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5 Chapter number 5

'Radio Spectrum Management: Policies, Regulations, Standards and Techniques' Mazar 2016, Chapter number 5 'RF Engineering and Link Budgets' Sections 5.4, 5.5 and 5.6. Updated on 25 October 2024

Contents

5.4Antennas: fundamental parameters

There is no RF signal without antennas. The antenna is a structure which transforms a guided electromagnetic wave on a transmission line to a plane wave propagating in free-space (transmission); it can be also served for reception (vice-versa). In the transmitting mode an RF signal source is applied to the input terminals of an antenna; the receiving antenna extracts electromagnetic energy and delivers it to a load. Antennas transmit to the free-space and receive from the free-space the modulated carrier signal at the radio frequency (RF). For the basic design, there is a complete reciprocity regarding reception and transmission; unlike feeding wires and compression, antennas are identical in all their characteristics as transmitting and receiving antenna: directivity, gain, radiation pattern, beamwidth, polarization, RF bandwidth, impedance and standing wave ratio; therefore, the same antenna may be used for transmitting and receiving.

Regulators may treat antennas as a tool for increasing spectral capacity. As an example, adaptive antennas provide significant link budget improvements and mitigate interference in communications systems; moreover, smart antennas may improve coverage and increase capacity in cellular systems. Multiple antennas are based on three fundamental principles (see Sesia, Toufik and Baker 20011:16, 251):

1) gain and spatial diversity, to improve robustness against multipath fading;
2) array gain, to concentrate the signal to one or more directions, in order to serve multiple users simultaneously, so called Multiple-Input and Multiple-Output MIMO[1] increase gain;
3) spatial multiplexing gain to transmit multiple signal streams to a single user. MIMO multiplexing does not expand bandwidth but necessitates added antennas and additional signal processing.

5.4.1Antenna: aperture, beamwidth, directivity and gain

The geometrics and electrics of antennas are detailed in the following figure. A reference to a rectangle aperture simplifies the explanation, but does not exclude elliptic, circulars and other antenna reflectors.

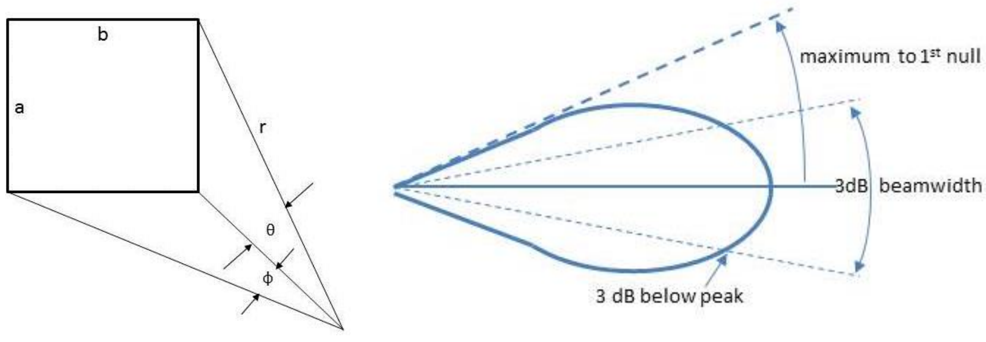

Figure 5.1 Antenna aperture (a) and beamwidths (b)

5.4.1.1Antenna directivity

The antenna directivity d (D in decibels) is defined (Balanis 2008 p.16) as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied".

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[1] MIMO enables multiple data input and multiple data output operation; multiple antennas send and receive more than one communication signal simultaneously. MIMO multiplies the performance of the signal, and is reflected in the two, three, or even more antennas found on the equipment

Directivity is defined in steradian[2](sr) by the beam solid angle Ω_A. Balanis (2008 p.18, formula 1.12) defines the maximum directivity d_0:

$$d_0 = \frac{4\pi}{\Omega_A(steradians)} \tag{5.4.1}$$

For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle Ω_A equals approximately the product of the antenna elevation θ and azimuth φ half power beamwidths (-3 dB) in radians[3] of the two perpendicular planes; see Lo YT and Lee SW 1988:1-29 and Balanis 1997 p.46.

$$d_0 = \frac{4\pi}{\Omega_A(steradians)} \approx \frac{4\pi}{\phi(\text{radian})\theta(\text{radian})} \tag{5.4.2}$$

As 2 π radians are equivalent to 360^0 (^{0} for degrees), using degrees instead of radians we get Kraus formula[4]:

$$d_0 = \frac{4\pi}{\Omega_\text{A}\,(\text{steradian})} \approx \frac{4\pi}{\phi(\text{radian})\theta(\text{radian})} \approx \frac{4\pi}{\phi^0\theta^0}\left(\frac{180}{\pi}\times\frac{180}{\pi}\right) = \frac{41,253}{\phi^0\theta^0} \tag{5.4.3}$$

5.1.1.1 Antenna gain

The antenna gain g (G in decibels) is defined (Balanis 2008:24) as "the ratio of the radiation intensity in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically. The radiation corresponding to the isotropically radiated power is equal to the total power accepted (input) by the antenna divided by 4π." In most cases the gain is relative to a lossless isotropic source; when the direction is not stated, the power gain is taken in the direction of maximum radiation.

The total efficiency of an antenna η is the ratio between the powers delivered to the antenna and the input power; it takes into account antenna losses such as mismatch with transmitter or receiver, and losses (conduction and dielectric). Following Balanis (2008 p.24, formulas 1.27 and 1.27a), η equals gain divided by directivity: $g(\theta,\varphi)= \eta\, d(\theta,\varphi)$. Inserting η we get:

$$g_0 = \eta d_0 = \eta\frac{4\pi}{\Omega_\text{A}\,(\text{steradian})} \approx \eta\frac{4\pi}{\phi(\text{radian})\theta(\text{radian})} \approx \eta\frac{4\pi}{\phi^0\theta^0}\left(\frac{180}{\pi}\times\frac{180}{\pi}\right) \approx \eta\frac{41,253}{\phi^0\theta^0} \tag{5.4.4}$$

Inserting a typical antenna efficiency of η=0.73 provides an approximate formula of the maximum gain g_0 for many practical antennas;

$$g_0 \approx \eta\frac{41,253}{\phi^0\theta^0} \approx 0.73\frac{41,253}{\phi^0\theta^0} \approx \frac{30,000}{\phi^0\theta^0} \tag{\textbf{5.4.5}}$$

Equation (5.4.5) appears at Balanis (2008 p.18, formula 1.30); ITU Handbook on Satellite Communications p. 104 (3bis) indicates that $g_0 = \dfrac{27,000}{\phi^0\theta^0}$.

When the antenna apertures a (elevation) and b (azimuth), see Figure 5.1 a, are expressed in the same unit as the wavelength λ, the antenna elevation θ and azimuth φ (-3 dB) beamwidth in radians are respectively:

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[2] Steradian is the standard unit of solid angle: corresponding surface area divided by square radius of the sphere.

[3] Radian is the standard unit of plane angles: the length of a corresponding arc divided by the circle radius.

[4] However, in this helpful equation, the spatial beam is generally elliptic and not rectangular, so the solid angle $\Omega_\text{A}\,(\text{steradian})$ is not exactly the product of the two orthogonal planes beamwidths φ (radian) and (radian) θ), and a geometric correcting factor is needed.

$$\varphi(\text{radian}) = (\lambda / a) \text{ and } \theta(\text{radian}) = (\lambda / b) \tag{5.4.6}$$

Inserting the half power beamwidths in equation

$$d_0 = \frac{4\pi}{\Omega_{A\,(\text{steradian})}} \approx \frac{4\pi}{\phi(\text{radian})\theta(\text{radian})} \approx \frac{4\pi}{(\lambda / a)(\lambda / b)} \approx \frac{4\pi(a \times b)}{\lambda^2} \tag{5.4.7}$$

5.1.1.2 Effective antenna aperture area

The overall antenna efficiency[5] η also equals the ratio of the radiated power divided by the input power, and the effective antenna aperture area A_e divided by the geometric aperture area A (Balanis 2008 p.24, formula 1.25, and 39 formula 1.53). To typify: for a rectangular aperture area A, the antenna surface equals $a \times b$ and the maximum effective area $A_e = \eta A = \eta\, a \times b$:.

$$\eta = \frac{P_{rad}}{P_{input}} = \frac{A_e}{A} = \frac{g_0}{d_0} \tag{5.4.8}$$

Combining equations (5.4.7) for d_o and (5.4.8) for η the maximum directivity equals:

$$d_0 = \frac{4\pi(a \times b)}{\lambda^2} = \frac{4\pi A}{\lambda^2} \tag{5.4.9}$$

Multiplying both sides at (5.4.9) by η, *setting* $g_0 = \eta d_0$, *and* $A_e = \eta A$

$$\eta d_0 = g_0 = \frac{4\pi\eta(a \times b)}{\lambda^2} = \frac{4\pi\eta A}{\lambda^2} = \frac{4\pi A_e}{\lambda^2} \tag{5.4.10}$$

Lo/Lee 1988:2-35 formula (95) and Cheng p. 532 formula (11-87) indicate $g = \frac{4\pi}{\lambda^2} A_e$; see equation (5.4.11). The antenna gain used for transmitting must be proportional to its effective area, exactly as used for receiving. The effective area is universal to receiving and transmitting antennas. Equation (5.4.11) can be derived from the reciprocity theorem: see Cheng:531-32. The effective antenna aperture area (see also ITU Handbook on Satellite Communications p. 45, equation 4):

$$A_e = \frac{g\lambda^2}{4\pi} \tag{5.4.11}$$

The above equation is most useful to develop the free–space propagation loss and conversion formulae; see next sections. For a fictitious no-loss isotropic radiator, whose electrical efficiency is 100%, with antenna gain g = 1, its receiving cross section (effective area) A_e averaged over all directions must be equal to $\lambda^2/4\pi$, the wavelength squared divided by 4π; see Lo/Lee 1988:2-35:

$$A_{e\ isotropic} = \frac{\lambda^2}{4\pi} \tag{5.4.12}$$

Useful formulas are derived from equations (5.4.1) and (5.4.11) for antenna efficiency η = 0.7, converting the (–3 dB) effective (θ and φ *multiplied by* $\sqrt{\frac{1}{\eta}}$) beamwidths angles θ_e and φ_e radians to degrees; using l as appropriate antenna length or diameter: l/λ (expressed in the

[5] Balanis 2008 p.23 determines total efficiency e_0 as a product of three dimensionless efficiencies: e_r reflection (mismatch), e_c conduction and e_d dialectric; e_{cd} (antenna radiation efficiency) = $e_c * e_d = p_{rad} / p_{input} = \eta = \frac{g_o}{d_o}$; $e_r = 1 - |\Gamma|^2$, where Γ is the insertion loss. In IEEE Standards, the gain does not include losses arising from impedance mismatch and from polarization mismatch. The author refers to the overall efficiency.

same unit)[6] may be estimated approximately by the beamwidths: $\sqrt{\frac{1}{\eta}}\frac{\lambda}{l}=\sqrt{\frac{1}{0.7}}\frac{\lambda}{l}=1.2\frac{\lambda}{l}$ and $l/\lambda \approx 1.2/beamwidth$(radians)

$$\varphi_e(\text{degrees}) = 1.2\frac{\lambda}{a}\times\frac{180}{\pi}\approx 70\frac{\lambda}{a} \text{ and}^7\ \theta_e(\text{degrees}) = 1.2\frac{\lambda}{b}\times\frac{180}{\pi}\approx 70\frac{\lambda}{b} \qquad (5.4.13)$$

Expressing the isotropically directivity and gain in decibels:

$$D_i = 10\log_{10} d \text{ and } G_i = 10\log_{10} g \qquad (5.4.14)$$

The gain g_d of a half-wave dipole equals 1.64; another expression used in practice is the gain relative to a half-wave dipole, G_d, that is:

$$G_d = G_i - 10\log 1.64 = G_i - 2.15 \qquad (5.4.15)$$

In decibels equation (5.4.5) looks (^{0} for degrees):

$$G_0 = 10\log 30{,}000 - 10log\theta^0 - 10log\varphi^0 = 44.8(\text{dBi}) - 10log\theta^0 - 10log\varphi^0 \qquad (5.4.16)$$

For circular antenna, where $\theta=\varphi$:

$$G_0 = 44.8 \text{ dBi} - 20\log\theta^0 \qquad (5.4.17)$$

For circular antennas, in cases where λ/l is not given, this ratio may be estimated; inserting $\theta(\text{degrees})\approx 70\frac{\lambda}{l}$ from (5.4.13), $G_0 = 44.8 \text{ dBi} - 20\log 70\frac{\lambda}{l}$.

Moreover, G_0 can be derived from $\frac{\lambda}{l}$:

$$G_0 = 7.9 - 20\log\frac{\lambda}{l} \qquad (5.4.18)$$

In RR AP8-10 and Recommendation ITU-R F.699 $20\log\frac{\lambda}{l} = G_0 - 7.7$, as g_o is approximated by $g_0 \approx \frac{28{,}800}{\phi^0\theta^0}$ instead of $g_0 \approx \frac{30{,}000}{\phi^0\theta^0}$, see equation (5.4.13); at Recommendation ITU-R F.1336 (equations 28a, 32a, 36 and 40), the semi-empirical relationship between gain and beamwidths of sectoral antenna is $g_0 = \frac{31{,}000}{\phi^0\theta^0}$.

5.4.2Three-dimensional radiation pattern and gain calculations

The radiation pattern represents the radiation properties of the antenna as a function of the angular coordinates. Antenna patterns are most important for analysis of interference and human hazards that enter mainly through the sidelobes. Several representations of a three-dimensional radiation pattern are possible: azimuthal radiation patterns at specific elevation angles, and vertical patterns at specific azimuthal angles are used to describe the full radiation pattern. The most important sections are the azimuthal patterns at the elevation angle at which the maximum antenna gain occurs and the vertical pattern at the azimuthal angle at the maximum antenna gain. These are referred to as the horizontal radiation pattern (HRP) and

[6] Equation (5.4.13) was added by an author's contribution to update ITU-R F.699 *recommends* 4.1.

[7] Handbook on Satellite Communications p. 49 equation 19 states $\theta(\text{degrees}) = 65\frac{\lambda}{a}$

the vertical radiation pattern (VRP), respectively. The following figure depict antenna patterns.

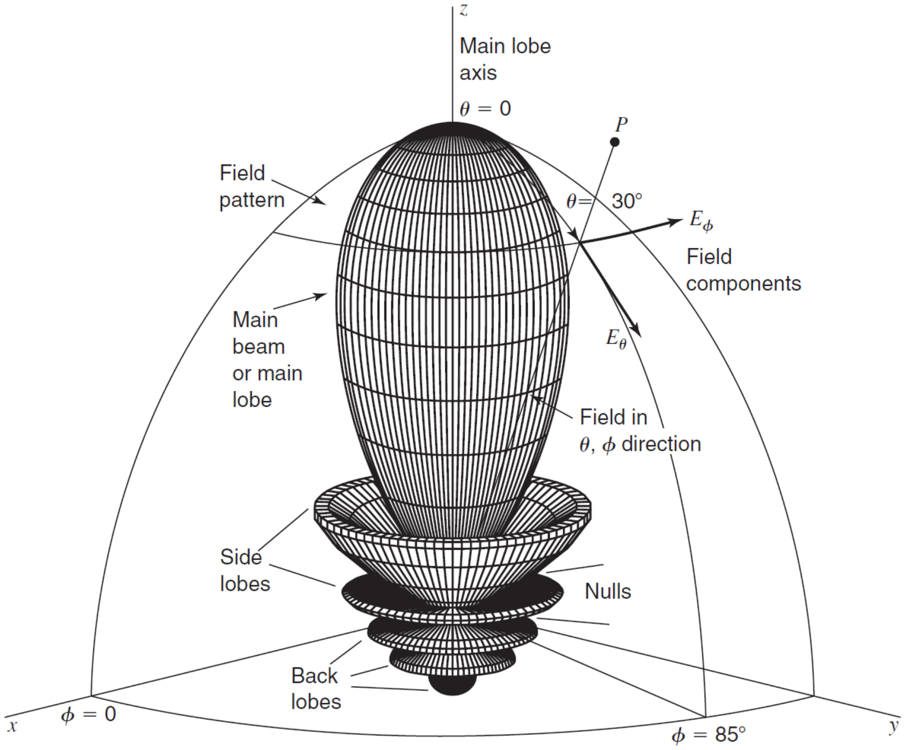

Figure 5.2 Three-dimensional amplitude field pattern (Balanis:6, fig 1.4)

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When the radiation from an antenna is in the far-field[8], i.e. when the distance from the antenna is such that its electromagnetic fields can be taken as being orthogonal to the direction of propagation, the antenna can be considered as a point source. In far-field conditions the power flux from a point source is always radial. The Poynting vector (see Recommendation ITU-R BS.1195 Fig. 2) and $\hat{E}_r$ (Balanis[9] p. 7 fig 1.4) result therefore only from two transverse electrical field components e_θ and e_φ (BS.1195) and ($\hat{E}_\theta$ and $\hat{E}_\phi$ in Balanis) as shown in Balanis Fig. 1.4; see **Error! Reference source not found.** (a).

In a spherical polar coordinate system the following parameters are denoted:

θ : elevation angle from the horizontal ($0 \le \theta \le \pi$ radians)

φ : azimuthal angle from the x-axis ($0 \le \theta \le 2\pi$ radians)

r : distance between the origin and the distant observation point

5.1.1.3 Calculating antenna directivity and gain

The antenna directivity d is defined as the ratio of its maximum radiation intensity to the radiation intensity (or power flux-density) of an isotropic source radiating the same total power; see BS.1195 and also Balanis (2008 p.18) (the directions of θ are different in Balanis and BS.1195); the maximum directivity d_0 can be expressed:

[8] The range of the observer point from the antenna is higher than 2 D^2/λ, where D is the largest dimension of the radiator (or the antenna diameter) and λ is the wavelength of the radio wave; see propagation, next section.
[9] ITU-R defines the elevation angle θ horizon to Zenith; Balanis and IEEE define θ Zenith to horizon.

$$d_o = \frac{4\pi \left|e(\theta,\varphi)\right|^2_{max}}{\int_0^{2\pi}\int_0^{\pi}\left|e(\theta,\varphi)\right|^2 \sin\theta \, d\theta \, d\phi} \tag{5.4.19}$$

where:

$e(\theta, \varphi)$: vector of the source electric field; expressed in spherical coordinate system;

$|e(\theta, \varphi)|$: magnitude of the electric field ; $|e(\theta, \varphi)| = \sqrt{\left|e_\theta(\theta,\varphi)\right|^2 + \left|e_\varphi(\theta,\varphi)\right|^2}$

Due to the law of conservation of energy, the total directivity integral equals 1. The following Equation (5.4.19) specifies the antenna directivity as a function only of the shape of the source radiation pattern. For a lossless isotropic source, by definition its antenna radiation $e(\theta,\varphi) \equiv 1$; setting $e(\theta, \varphi)=1$ in equation (5.4.19):

$$d_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi}\sin\theta \, d\theta \, d\phi} = \frac{4\pi}{\int_0^{2\pi}\left[-\cos(\pi)+\cos(0)\right]d\phi} = \frac{4\pi}{2\int_0^{2\pi} d\phi} = \frac{4\pi}{4\pi} = 1$$

Dividing directivity d by the maximum directivity d_0 merely normalizes the directivity (and radiation intensity) and it makes its maximum value unity. The relative antenna pattern as a function of θ and φ equals d/d_0 and is derived from equation (5.4.19):

$$\frac{\dfrac{4\pi\, e(\theta,\varphi)^2}{\int_0^{2\pi}\int_0^{\pi}\left|e(\theta,\varphi)\right|^2 \sin\theta \, d\theta \, d\phi}}{\dfrac{4\pi\left|e(\theta,\varphi)\right|^2_{max}}{\int_0^{2\pi}\int_0^{\pi}\left|e(\theta,\varphi)\right|^2 \sin\theta \, d\theta \, d\phi}} = \frac{e(\theta,\varphi)^2}{\left|e(\theta,\varphi)\right|^2_{max}} \tag{5.4.20}$$

5.1.1.4 Calculating antenna patterns and sidelobes

As already mentioned, antenna patterns are significant in analysing co-sharing and RF human hazards. In the absence of particular information concerning the radiation pattern of the antenna, simulations are needed.

5.1.1.4.1 Patterns derived directly from distribution (excitation) of field-strength

We may consider the antenna current distribution a function of either the elevation or azimuth coordinates. The directivity pattern, $F(\mu)$, of a given distribution is found from the finite Fourier transform, see Recommendation ITU-R M.1851:

$$F(\mu) = \frac{1}{2}\int_{-1}^{+1} f(x)\cdot e^{j\mu x} dx \tag{5.4.21}$$

Where, $f(x)$ is the relative shape of field distribution, and $\mu = \pi\left(\frac{l}{\lambda}\right)\sin(\theta)$;

l: length of aperture; λ: wavelength;

θ: angle relative to aperture normal, when the pointing angle (main beam) is zero;

x: normalized distance along aperture $-1 \le x \le 1$; j: complex number notation.

Important to note, that the distribution (excitation) of field-strength along the aperture delineates the antenna patterns (the sidelobes) in elevation and azimuth. As the directivity and gain are derived from the finite Fourier transform, smaller apertures provide higher antenna main lobes[10]. The patterns are valid in ±π/2 (±90°) from beam scan angle relative to antenna boresight. The parameters and formulae in Table 5.1 are correct only in the case where the field amplitude at the edge of the antenna aperture is equal to zero, and stays within the bounds of the main lobe and first two antenna side lobes. If real antenna patterns are available, then those should be used.

The shaping of the field-strength increases the angle of the main beam and decreases the sidelobes[11]; lower sidelobes level is an important feature to improve RF sharing and decrease vulnerability from jammers and RF interference. For $f(x)$ (where $-1 \le x \le 1$) and directivity pattern $F(\mu)$, Table 5.1[12] details the θ_3 half power beam-width (radians and degrees) and first side-lobe level below main lobe peak (dB).

Table 5.1 Antenna directivity parameters

Relative shape of field distribution *f*(*x*)	**Directivity pattern *F*(μ)**	**θ_{3dB} beam-width[13]**		**μ as a function of θ_{3dB} (θ_{3dB}, radians)**	**First side-lobe (dB)**
		(radians)	**(degrees)**		
Uniform value of 1	$\frac{\sin(\mu)}{\mu}$	$0.89\left(\frac{\lambda}{l}\right)$	$50.8\left(\frac{\lambda}{l}\right)$	$\frac{\pi\cdot 0.89\cdot\sin(\theta)}{\theta_{3dB}}$	−13.2
COS(π*x/2)	$\frac{\pi}{2}\left[\frac{\cos(\mu)}{\left(\frac{\pi}{2}\right)^2-\mu^2}\right]$	$1.2\left(\frac{\lambda}{l}\right)$	$68.8\left(\frac{\lambda}{l}\right)$	$\frac{\pi\cdot 1.2\cdot\sin(\theta)}{\theta_{3dB}}$	−23
COS^2 (π*x/2)	$\frac{\pi^2}{2\cdot\mu}\left[\frac{\sin(\mu)}{(\pi^2-\mu^2)}\right]$	$1.45\left(\frac{\lambda}{l}\right)$	$83.2\left(\frac{\lambda}{l}\right)$	$\frac{\pi\cdot 1.45\cdot\sin(\theta)}{\theta_{3dB}}$	−32
COS^3 (π*x/2)	$\frac{3\cdot\pi\cdot\cos(\mu)}{8}\left[\frac{1}{\left(\frac{\pi}{2}\right)^2-\mu^2}-\frac{1}{\left(\frac{3\cdot\pi}{2}\right)^2-\mu^2}\right]$	$1.66\left(\frac{\lambda}{l}\right)$	$95\left(\frac{\lambda}{l}\right)$	$\frac{\pi\cdot 1.66\cdot\sin(\theta)}{\theta_{3dB}}$	−40
COS^4 (π*x/2)	$\frac{3\pi^4\sin(\mu)}{2\mu(\mu^2-\pi^2)(\mu^2-4\pi^2)}$	$1.85\left(\frac{\lambda}{l}\right)$	$106\left(\frac{\lambda}{l}\right)$	$\frac{\pi\cdot 1.85\cdot\sin(\theta)}{\theta_{3dB}}$	−47

The following Figure depicts the relative shapes of the field distribution functions $f(x)$: uniform value of 1, cos(π*x/2), $\cos^2$(π*x/2), $\cos^3$(π*x/2) and $\cos^4$(π*x/2).

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[10] It can be compared to the wireless signal in the frequency-domain transformed from the time domain: smaller pulses in time provide higher RF bandwidths.

[11] The law of conservation of energy decrees that more energy in the main beam (larger beam-width) is compensated by less energy in the sidelobes.

[12] Table 5.1 uses the frame of M.1851 Table 2; the row $\cos^4$(π*x/2) and column of radians do not appear.

[13] The values in this column can be roughly compared with equation (5.4.13)

$\theta(\text{degrees}) = 1.2\frac{\lambda}{l}\times\frac{180}{\pi}\approx 70\frac{\lambda}{l}$; or in radians $\theta = 1.2\frac{\lambda}{l}$.

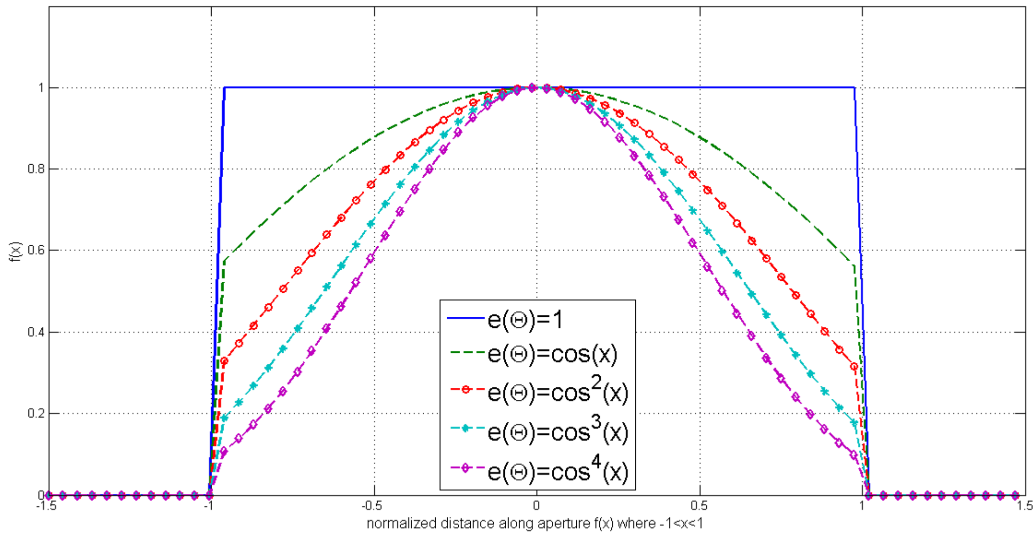

Figure 5.3 Relative shapes of the field distribution functions

The following Figure depicts the five patterns[14], derived from the different relative shapes uniform $f(x)$ of the field-strength distribution functions e, along the aperture.

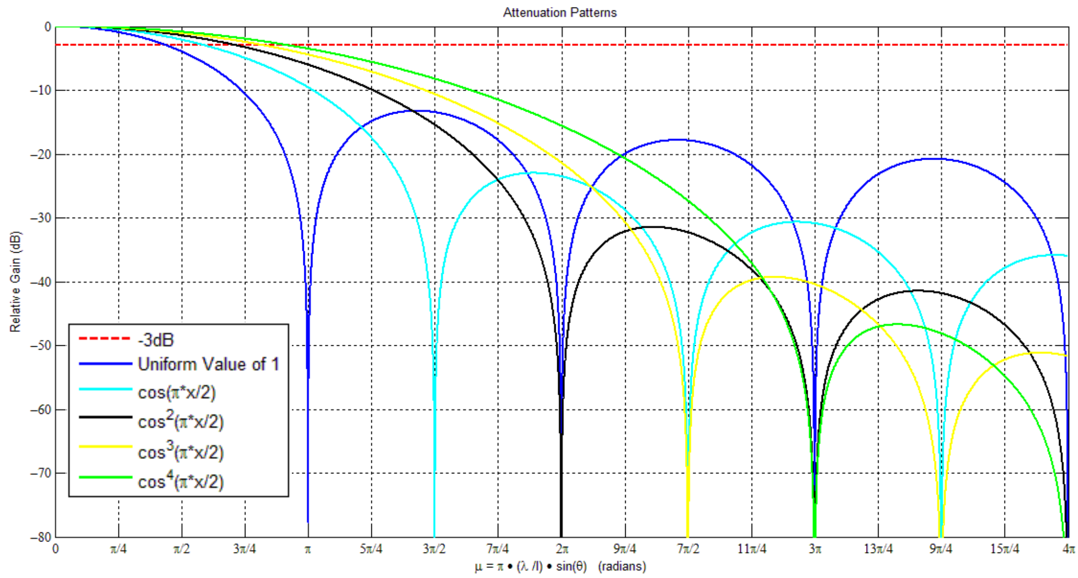

Figure 5.4 Antenna attenuation patterns with different distribution functions

The dashed red is the 3 dB point; the green is $\cos^4(\pi x/2)$, yellow is $\cos^3(\pi x/2)$, black is $\cos^2(\pi x/2)$, light blue is $\cos(\pi x/2)$ and dark blue is uniform.

The uniform illumination case is interesting, x (normalized distance along aperture) equals 1 in $-1 \le x \le 1$ and 0 outside. Following equation (5.4.20), when the rectangular reflector is illuminated uniformly $e(\theta,\varphi)=1$, and for $\mu=\pi\left(\frac{l}{\lambda}\right)\sin(\theta)$:

$$F(\mu)=\frac{1}{2}\int_{-1}^{+1} f(x)\cdot e^{j\mu x}dx=\frac{1}{2}\int_{-1}^{+1} e^{j\mu x}dx=\frac{1}{2j\mu}(e^{j\mu}-e^{-j\mu})=\frac{(\cos\mu+j\sin\mu)-[(\cos(-\mu)+j\sin(-\mu)]}{2j\mu}=\frac{\cos\mu+j\sin\mu-\cos\mu+j\sin\mu}{2j\mu}=\frac{\sin\mu}{\mu}$$

The antenna pattern, a spatial Fourier transform, converts two orthogonal θ (elevation) and φ

[14] M.1851 Fig. 2 shows 4 (not 5) patterns, normalized to a beam-width of 8^0 (0.14 radians). The Author provided to ITU-R Study Group 5 this Figure

(azimuth) pulse waves ('1' inside, '0' outside the rectangular) to two Sinc[15] functions (sinθ/θ) and (sinφ/φ), respectively. The following figure is the numerical attenuation pattern for the uniform distribution on rectangular reflector; the three-dimensional isometric projec pattern depicts off boresight (axes in radians) absolute relative attenuation.

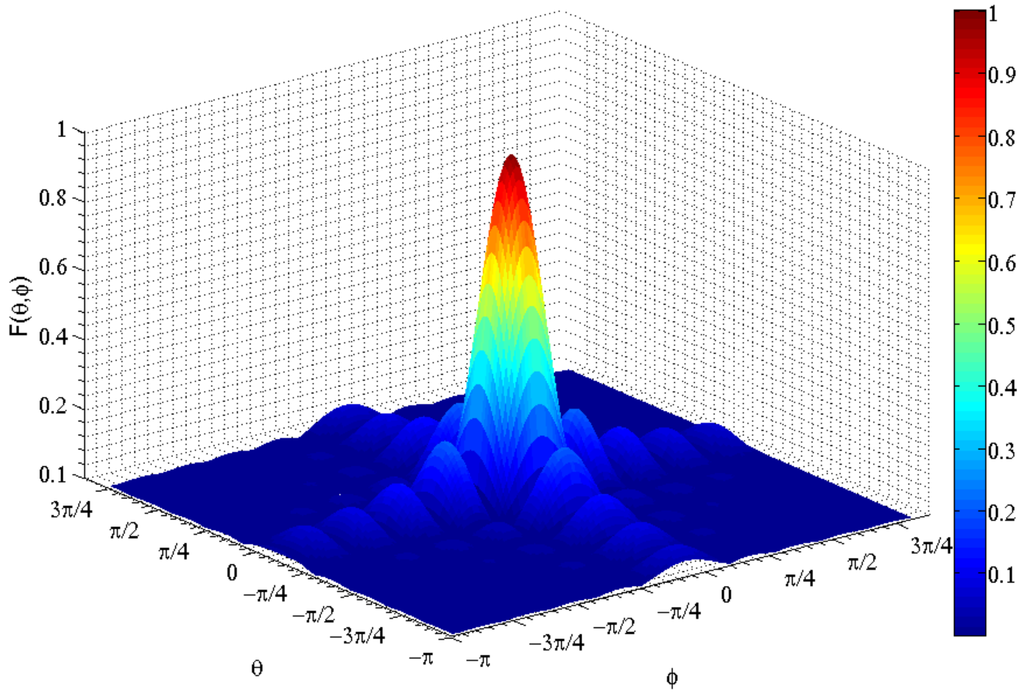

Figure 5.5 Matlab 3dimensions antenna attenuation pattern of rectangular reflector

5.4.1.4.2Patterns derived from Recommendations ITU-R F.1336 and F. 699

Where orthogonal off-boresight angles are φ in azimuth and θ in elevation, the equivalent off-axis angle ψ =arcos (cos φ x cos θ), $0^0 \leq \psi \leq 180^0$; see Rec. ITU-R F.1336 equations 41b and 2a4. If the reference radiation pattern is assumed to be rotationally symmetric about the boresight axis, the angle ψ instead of φ and θ may be used, to calculate the antenna pattern in Rec ITU-R F.699 and the power of cosines. To exemplify: for off-boresight angles $\varphi=5^0$ and $\theta=6^0$ ψ= arcos (cos5 x cos6) =7.8^0.

Recommendation ITU-R F.699 is most useful to provide reference patterns[16]. Important to note that F.699 provides antenna gain, in contrast to M.1851, that specifies antenna attenuation, relative to the maximal gain. The F. 699 pattern is calculated based only on the antenna length or diameter l, expressed in the same unit as the wavelength λ. Given that the gain of the first sidelobe G_1 (dBi), these formulas apply:

1) For frequencies between 1 GHz to about 70 GHz, where the l/λ ratio is greater than 100, the following equations should be used.

[15] Sinc 'square' as the directivity and attenuation pattern in the far-field depend on the square of field-strength.
[16] Some of F. 699 equations are also used to assess interference between systems in the fixed satellite service (FSS), and between earth stations in FSS and stations of other services. Same equations as F. 699 appear at RR AP8-10 and in Appendix 4 to Annex 1 at Recommendation ITU-R P.620; similar equations are found at Rec. ITU-R S.465 and Report S.2196. For the l/λ ratio greater than 100, Rec ITU-R F.1245 quotes F.699; F.758 also refers to F.699.

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3}\left(\frac{l}{\lambda}\varphi\right)^2 \quad \text{for} \quad 0° < \varphi < \varphi_m$$

$$G(\varphi) = G_1 \quad \text{for} \quad \varphi_m \le \varphi < \varphi_r$$

$$G(\varphi) = 32 - 25\log\varphi \quad \text{for} \quad \varphi_r \le \varphi < 48°$$

$$G(\varphi) = -10 \quad \text{for} \quad 48° \le \varphi \le 180°$$

where:

$G(\varphi)$:Gain relative to an isotropic antenna

φ :off-axis angle (degrees)

G_1 :Gain of the first sidelobe $= 2 + 15\log\frac{l}{\lambda}$

$$\varphi_m\,(\text{degrees}) = \frac{20\lambda}{l}\sqrt{G_{max} - G_1}$$

$$\varphi_r\,(\text{degrees}) = 15.85\left(\frac{l}{\lambda}\right)^{-0.6}$$

2) For frequencies between 1 GHz to about 70 GHz, where the l/λ ratio is less than or equal to 100 the following equations should be used:

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3}\left(\frac{l}{\lambda}\varphi\right)^2 \quad \text{for} \quad 0° < \varphi < \varphi_m$$

$$G(\varphi) = G_1 \quad \text{for} \quad \varphi_m \le \varphi < 100\frac{\lambda}{l}$$

$$G(\varphi) = 52 - 10\log\frac{l}{\lambda} - 25\log\varphi \quad \text{for} \quad 100\frac{\lambda}{l} \le \varphi < 48°$$

$$G(\varphi) = 10 - 10\log\frac{l}{\lambda} \quad \text{for} \quad 48° \le \varphi \le 180°$$

3) For frequencies in the range[17] 100 MHz to less than 1 GHz, in cases where the l/λ ratio is greater than 0.63 (G_{max} is greater than 3.7 dBi), the following equations should be used:

$$G(\varphi) = G_{max} - 2.5 \times 10^{-3}\left(\frac{l}{\lambda}\varphi\right)^2 \quad \text{for} \quad 0° < \varphi < \varphi_m$$

$$G(\varphi) = G_1 \quad \text{for} \quad \varphi_m \le \varphi < 100\frac{\lambda}{l}$$

$$G(\varphi) = 52 - 10\log\frac{l}{\lambda} - 25\log\varphi \quad \text{for} \quad 100\frac{\lambda}{l} \le \varphi < \varphi_s$$

$$G(\phi) = -2 - 5\log\frac{l}{\lambda} \quad \text{for} \quad \varphi_s \le \varphi \le 180°$$

where: $\varphi_s = 144.5\left(\frac{l}{\lambda}\right)^{-0.2}$

The following Figure depicts the azimuth antenna pattern of a rectangular antenna, calculated by ITU-R F. 699 equations 3) (as RF is smaller than 1,000 MHz), for: RF 514 MHz, λ=300/514=0.58m; G_{max}=37 (dBi); λ/l =0.1; azimuth beam-width $\varphi_{3db}= 7^0$ and $70\lambda/l=7^0$.

[17] In order to use these equations to coordinate the forthcoming digital TV and the fixed service, the author contributed to ITU-R between 2001 to 2024; see http://mazar.atwebpages.com/ContributionstoITU.html

For this specific case: G_1: the Gain of the first sidelobe=2+15log$\frac{l}{\lambda}$=2+15=17(dBi);

$\varphi_m(\text{degrees}) = \frac{20\lambda}{l}\sqrt{G_{max} - G_1} = \frac{20}{10}\sqrt{37-17} \approx 9^0$ and

$\varphi_s = 144.5\left(\frac{l}{\lambda}\right)^{-0.2} = 144.5x10^{-0.2} \approx 91^0$, therefore:

$$G(\varphi) = G_{max} - 2.5\times10^{-3}\left(\frac{l}{\lambda}\varphi\right)^2 = 37 - 0.25\,\varphi^2 \quad \text{for} \quad 0° < \varphi < 9^0$$

$$G(\varphi) = G_1 = 17\ (\text{dBi}) \quad \text{for} \quad \varphi_m \le \varphi < 100\frac{\lambda}{l} = 10^0$$

$$G(\varphi) = 52 - 10\log\frac{l}{\lambda} - 25\log\varphi = 42 - 25\log\varphi \quad \text{for} \quad 10^0 \le \varphi < 91^0$$

$$G(\phi) = -2 - 5\log\frac{l}{\lambda} = -7\ (\text{dBi}) \quad \text{for} \quad 91^0 \le \varphi \le 180°$$

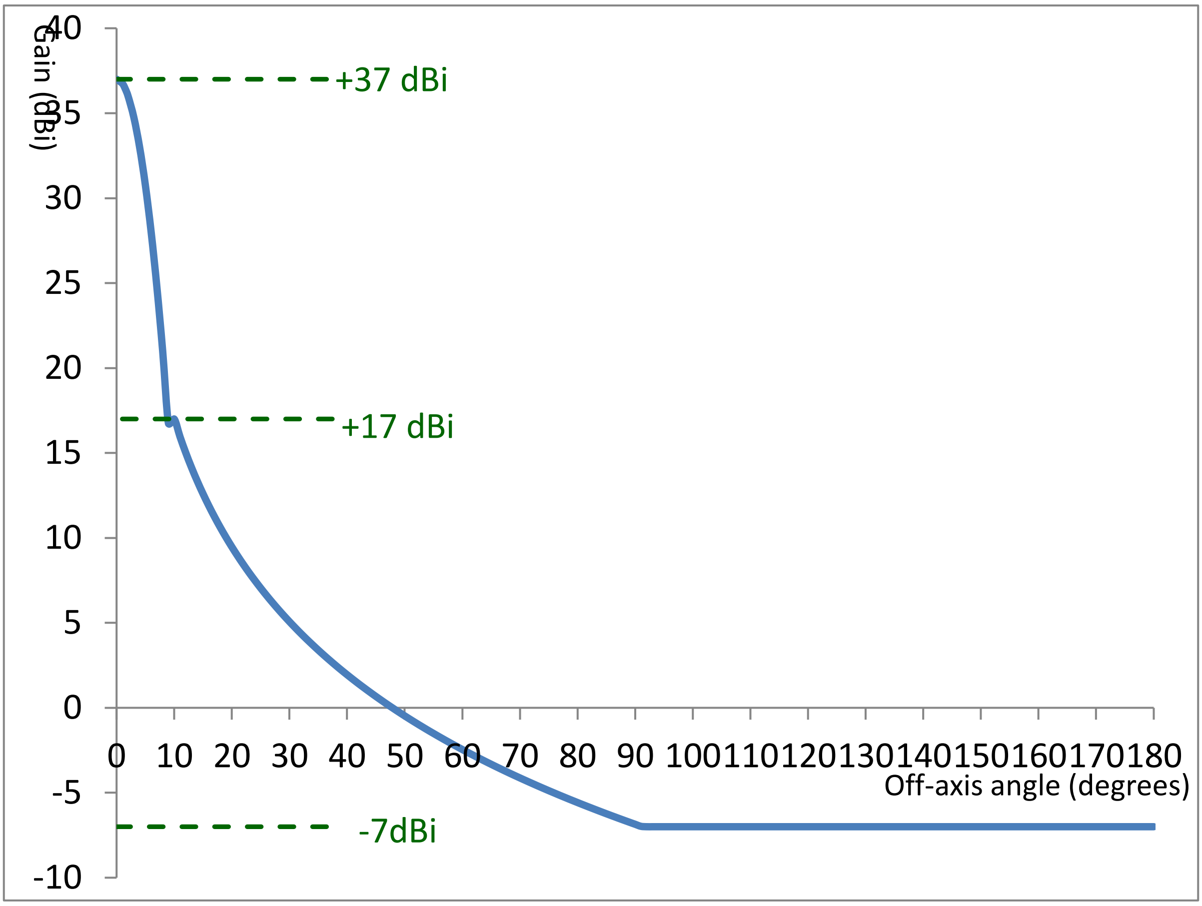

Figure 5.6 Horizontal pattern of 37 dBi antenna calculated by ITU-R Rec. F.699

5.4.1.43Pattern approximation by power of cosines

Many aperture-type antennas have a single major lobe and the backward is negligible; their far-field patterns can be approximated by simple and useful analytical functions; see Lo YT and Lee SW 1988 pp.1-28 and Balanis 1997 p.48. The normalised/relative numeric gain for elevation (el) $0\leq \theta \leq 2\pi$ and azimuth (az) $0\leq\varphi\leq2\pi$ is equal:

$$g(\theta)=\left|(\cos\theta)^{q_{el}}\right| \text{ and } g(\varphi)=\left|(\cos\varphi)^{q_{az}}\right| \tag{5.4.22}$$

At the half-power angles (antenna beamwidths) ½ θ_{3db} and ½ φ_{3db}, the numeric gains $g(\theta)$ and $g(\varphi)$ equal 0.5; therefore, the exponents q_{el} and q_{az} can be calculated:

$$g(\frac{1}{2}\theta_{3db})=\cos^{q_{el}}(\frac{1}{2}\theta_{3db})=0.5 \text{ and } g(\frac{1}{2}\varphi_{3db})=\cos^{q_{az}}(\frac{1}{2}\varphi_{3db})=0.5$$

$$q_{el}=\frac{\log 0.5}{\log(\cos\frac{1}{2}\theta_{3db})} \text{ and } q_{az}=\frac{\log 0.5}{\log(\cos\frac{1}{2}\varphi_{3db})} \tag{5.4.23}$$

q_{el} equals q_{az} when elevation and azimuth beamwidths are equal, e.g. dish antennas.
Actually, cosine patterns are the envelope of the antenna sidelobes; for $0\leq|\theta|\leq\pi/2$, $g(\theta)=(\cos\theta)^{q_{el}}$ decreases with $|\theta|$, as $g'(\theta)=\frac{dg}{d\theta}=-\sin\theta\times q_{el}\times(\cos\theta)^{q_{el}-1}$ is negative for positive θ and $g'(\theta)$ is positive for negative θ [18].
Cosine patterns are useful; e.g. to simulate the elevation cellular base station antennas see Recommendation ITU-T K.52 p. 35, Dariusz 2003 p.42 and Linhares, Terada and Soares 2013 p.148; source Figure 5 in Linhares, Terada and Soares 2013 and Figure 4.5 in Linhares 2015 Doctorate thesis[19], depicts reliable approximation; it compares the model to the real pattern of Kathrein 742 265 antenna with θ_{3db} half-power beamwidth 10º, resulting q = 181.8062, and electric tilt 7º. The deviation of the $\cos^q(\theta-\alpha)$ model from the real antenna until the half-power angle is less than 2%, and until the relative gain of -6 dB (0.25 in linear scale) is less than 7%. The relative deviation beyond -6 dB attenuation increases; the simulation is fairly accurate also above the first null, at $\theta >2.26$ (½ θ_{3dB}), to approximate the sidelobe level.

[18] Like at the uniform illumination of the reflector, where the normalized $g(\varphi)$ equals $\left(\frac{\sin\varphi}{\varphi}\right)^2$,
$g(\varphi)=(\cos\varphi)^{q_{az}}$ decreases at $0<\varphi<\pi$, as the $g'(\varphi)=\frac{dg}{d\varphi}=-\sin\varphi\times q_{az}\times(\cos\theta)^{q_{az}-1}$ is negative; $g(\varphi)$ increases at $-\pi<\varphi<0$, as $g'(\varphi)$ is positive;

[19] In Portuguese: *Contribuições ao estudo da exposição humana a campos eletromagnéticos na faixa de radiofrequências*

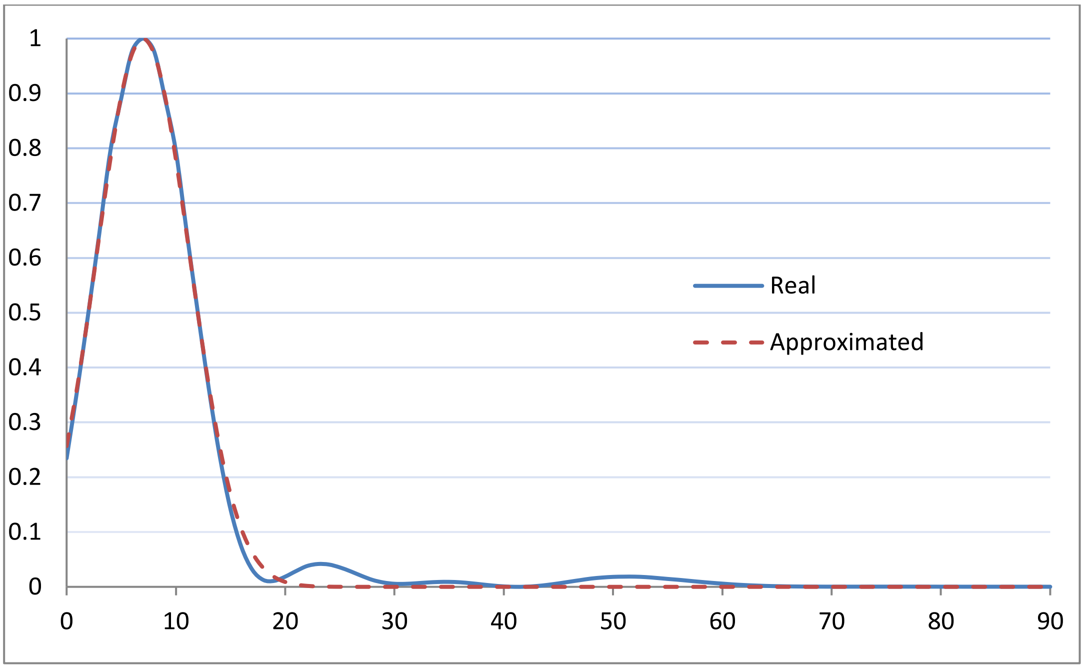

Figure 5.7 Real elevation pattern versus calculated $\cos^n(\theta-\alpha)$ for a tilt of 7^0

Disclaimer: reproduced with permission Dr. Agostinho Linhares.

θ_{3db} can be calculated from (5.4.1.3), using the same dimensions for λ wave length and b aperture (elevation or azimuth) $\theta(\text{3db, degrees}) = 1.2\frac{\lambda}{b} \times \frac{180}{\pi} \approx 70\frac{\lambda}{b}$. By determining, the absolute value at $g(\theta) = \left|(\cos\theta)^{q_{el}}\right|$ is not needed. For $g(\theta) = (\cos\theta)^{2N}$, $\theta_{3db} = 2\cos^{-1}\left(0.5^{1/2N}\right)$, see equation 33 at Rec ITU-R F.1336.

5.4.3 Antenna polarization, bandwidth, insertion loss and impedance

5.4.3.1 Polarization and bandwidth

Polarization is the orientation of $\vec{e}$, the electrical field vector, which may be in a fixed direction or may change in time. The antenna polarization is defined by the polarization vector of the transmitted wave in the far-field. The different forms of wave polarization (linear and circular) are special cases of the more general case of elliptical polarization.
Figure 5.8 Elliptical polarization

[20] depicts an elliptically polarized wave; the signal is represented by two mutually perpendicular linear waves, propagating along the z-axis and having the respective electrical fields expressed by:

$$e_x = e_1 sin(\omega t) \text{ and } e_y = e_2 sin(\omega t + \phi) \qquad (5.4.24)$$

$e_x = e_1 \sin \omega t$; $e_y = e_2 \sin (\omega t+\phi)$, where ϕ is the phase difference between the two waves. When the elliptically polarized wave travels along the z-axis, the resulting $\vec{e}$ vector describes an ellipse whose semi-axes are given by e_1 and e_2.

[20] See Balanis 2008 p.219 Fig. 5.21 and Recommendation ITU-R BS.1195 Fig. 4.

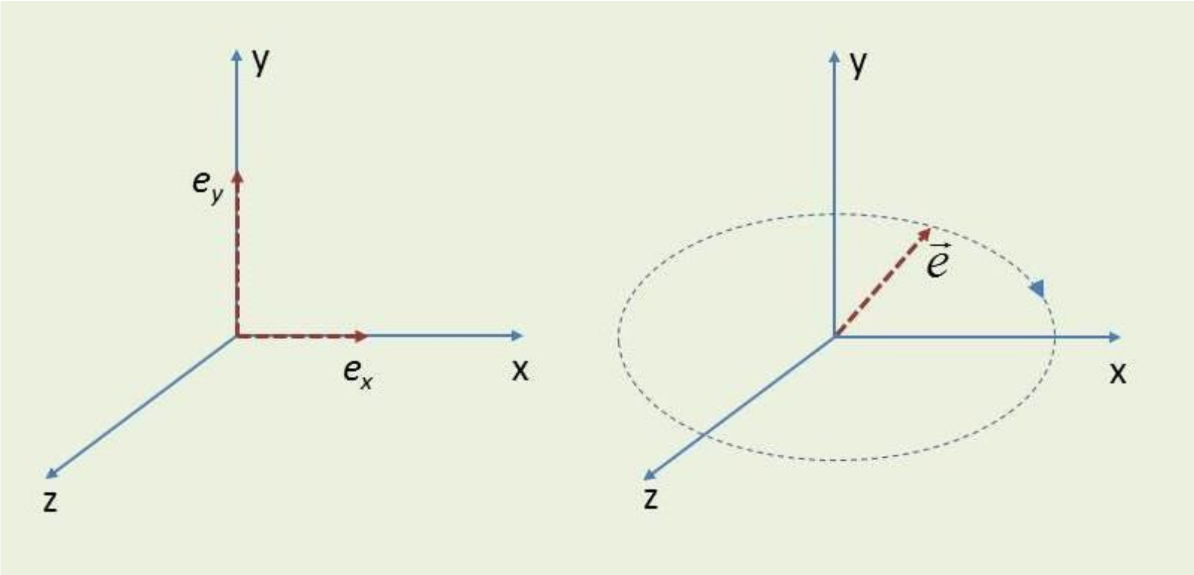

Figure 5.8 Elliptical polarization

The linear and circular polarizations are special cases of the elliptical polarization. Horizontal and vertical polarizations occur when either $e_y = 0$ (horizontal polarization) or $e_x = 0$ (vertical polarization). A 45° π/4 slant polarization occurs when in equation (5.4.21) $e_1 = e_2$ and φ = 0. Circular polarization occurs $e_y = e \sin wt$ and $e_x = +- e \cos wt$. When the sign is positive, the rotation of the wave is clockwise in the positive z-axis direction (right-hand circular polarization); when the sign is negative, left-hand circular polarization occurs.
The polarization vector is the normalized phasor of the electric field vector; It is a complex-valued vector of unit magnitude $a(t) = 1$; therefore the I and Q components may also represent the polarization of an electromagnetic wave: x and y are the field-strength directions of a plane wave propagating along the z axis; the polarization phase (or tilt angle) equals *arctan* (y/x).
Two **orthogonal polarizations** (such as vertical/horizontal, ±π/4 slanted, left/right hand) provide almost uncorrelated signals in a fading environment. Cross-polarization reception enhances spectral efficiency as it mitigates interference in wireless systems; polarization diversity may increase the wanted signal. Polarization diversity[21] achieves similar results as spaced antennas, with a single antenna.
Balanis (2008:26) defines the **antenna bandwidth** as 'the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard'. Those parameters may shape the bandwidth: input impedance, gain, radiation pattern, beamwidth, polarization, sidelobe level, radiation efficiency, etc. For f_o center frequency, the bandwidth for broadband antenna FBW_{bb} and narrowband FBW_{nb} equal:

$$FBW_{bb} = \frac{f_{max}}{f_{min}} \text{ , } FBW_{nb} = \frac{f_{max} - f_{min}}{f_0} 100\% \qquad (5.4.25)$$

[21] Two arrays of receiving cross-polarized base station antenna improve the uplink cellular signal up to 6 dB; this receiver polarization diversity also decreases the link un-balance, derived from higher downlink transmitting power, relative to the handset uplink power.

5.4.3.2 Antenna Insertion loss and Impedance

The antenna input impedance is defined as the ratio of the voltage to current at its terminals; this impedance is frequency dependent. If the antenna is not matched to the input signal, a standing wave is induced along the transmission line. The ratio of the maximum voltage to the minimum voltage along the line is the voltage standing wave ratio (VSWR).

For a forward wave voltage v_f and reflected wave voltage v_r, the reflection coefficient equals:

$\Gamma = \frac{v_r}{vf} = \sqrt{\frac{p_r}{p_f}}$; absolute value of Γ is $|\Gamma| = \rho$. $v_{max} = v_f + v_r$ and $v_{min} = v_f - v_r$.

We define VSWR:

$$vswr = \frac{v_{\max}}{v_{\min}} = \frac{1+\rho}{1-\rho} \text{ and } \rho = \frac{vswr - 1}{vswr + 1} \qquad (5.4.26)$$

The return loss (RL) =20 log ρ=20 log (VSWR-1) - 20 log (VSWR+1); the mismatch loss (ML) is the ratio of incident power to the difference between incident and reflected power; ML (dB) = -10 log (1- ρ^2).

5.6 Propagation

5.6.1 General

The attenuation of the RF signal is fundamental to understand RF environment; it is the relation between the transmitted to the received signal power. Realistic modeling of propagation characteristics is essential to assess radio coverage and interference. It is much convenient to compare the propagation losses to the attenuation in free-space. There is a need of free-space (emptiness) between transmitter and receiver, in order to avoid attenuations due to obstruction and to gas. However, the ground reflects waves 10 kHz to 30 MHz (see ITU 2014 Handbook on Ground wave propagation) to enable longer propagation. The ground wave signal amplitude depends on range and the electrical characteristics of the ground; also the signal amplitude does not remain constant for small changes in location (from several hundred metres). Recommendation ITU-R P.368 Fig. 1-11 contains field-strength curves as a function of distance with RF as a parameter, and for different conductivity σ permittivity ε. Moreover, long range propagations 2 to 30 MHz depend on ionospheric reflections; the ionosphere is transparent for µwaves but reflects HF waves. The ionospheric layers (D, E, F1, and F2) are at various heights 50 – 300 kM. Over-horizon communication range is several thousand Km, but it suffers from fading. The reflectivity depends on time, frequency of incident wave, electron density, solar activity, etc.; it is difficult to predict with precision. The U.S. National Telecommunications and Information Administration (NTIA) Institute for Telecommunication Sciences (ITS), and its Library (Boulder Colorado) may provide useful information on National Bureau of Standards NBS Standard 101. The basic model also for the ionospheric propagation is the free-space.

The main elements influencing the propagation loss between transmitter and receiver are (see Recommendation ITU-R P.1812) line-of-sight, diffraction (embracing smooth-earth, irregular terrain and sub-path cases), tropospheric scatter, anomalous propagation (ducting and layer reflection/refraction), variation in clutter, location variability, building entry losses and average earth radius[22] =6,371 km. There are many propagation models (such as, Longley and Rice model, Hata Okumura, Walfish Ikegami etc.); this chapter will elaborate the free-space model which is the basis of all models. Enclosed a useful link to retrieve the digital terrain elevation data (DTED); the link to meteorological parameters:.

5.6.2 Friis transmission equation and free-space propagation loss- power

Using the international system of units (SI):

p_t = transmitter output power	(Watts)
g_t = transmitter antenna gain	(dimensionless, with no units)
d =observation distance from transmitter to receiver	(m)
pd = incident power-density at the receiver	(W/m^2)
A_e = effective area of receiver's antenna	(m^2)
λ = wave length	(m)
g_r = receiver antenna gain	(dimensionless)
p_r = received power	(Watts)
p_l = propagation loss	(dimensionless)

[22] For flat plan, the curvature of the earth (earth radius) determines the maximal line-of-sight distance, between transmitter and receiver antennas. The declining pressure of the atmosphere with height 'bends' the radio waves towards earth, and actually increases the earth radius by a factor around 4/3 , to provide earth radius a_e, the notional effective Earth radius (m) of circa 8,500 km. For self-consistent units, altitudes h_1 and h_2 above ground level, the distance of line-of-sight d_{los} over flat earth (no topography or other obstacles):

$$d_{los} = \sqrt{2a_e}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

The received signal is linearly related to the transmitted power, propagation loss and the antenna gains (of transmitter and receiver); losses from the transmitter to its antenna and from the receiver's to its antenna attenuate the signal. In the free-space far-field (see later) the power-density derived from a transmitter p_t and antenna gain g_t at distance d equals:

$$pd = \frac{p_t g_t}{4\pi d^2} \tag{5.4.27}$$

The received power p_r equals the received power-density pd multiplied by the effective area A_e: $p_r = \frac{p_t g_t}{4\pi d^2} \times A_e$. Using (5.4.11) to replace A_e by $\frac{g_r \lambda^2}{4\pi}$, we get: $p_r = \frac{p_t g_t}{4\pi d^2} \times \frac{g_r \lambda^2}{4\pi}$.

$$p_r = \frac{p_t g_t}{4\pi d^2} \times \frac{g_r \lambda^2}{4\pi} \tag{5.4.28}$$

The *Friis* (named after Harald T. Friis) *transmission equation* relates the power delivered to the receiver antenna p_r to the input power of the transmitting antenna p_t.. Expressing p_r and p_t in the same units, the *Friis transmission equation* expressed numerically looks:

$$\frac{p_r}{p_t} = \frac{\frac{p_t g_t}{4\pi d^2} \times \frac{g_r \lambda^2}{4\pi}}{p_t} = g_t \times g_r \left(\frac{\lambda}{4\pi d}\right)^2 \tag{5.4.29}$$

The term $\left(\frac{4\pi d}{\lambda}\right)^2$ is independent of antenna gains; it is called the free-space loss factor. Where d (distance) and λ (wave length) are expressed in the same unit, the *free-space* basic propagation loss p_l (see Recommendation ITU-R P.525) looks:

$$p_l = \left(4\pi d / \lambda\right)^2 \tag{5.4.30}$$

As range λ(wavelength) is the product of c_0(velocity) and time t(cycle period =1/f) , $\lambda \equiv c_0 \text{x} t$, c_o(velocity of light)≡λxf; see equation (4.5.30)- another way to derive λ.

The free-space path loss equals: $p_l = \left(\frac{4\pi d}{\lambda}\right)^2 = \left(\frac{4\pi df}{c_0}\right)^2$

The free-space path loss expressed logarithmically by wavelength or frequency:

$$P_l\left(\text{dB}\right) = 20\ log\left(4\pi d / \lambda\right) = 20\ log\left(4\pi df / c_0\right) = 20\ log\left(4\pi df\right) - 20\ log c_0 \tag{5.4.31}$$

The velocity of light in vacuum c_0≡ 299 792 458 m/s (see BIPM 2006 p. 112) ≈ $300 \text{x} 10^6$ m/s, $\lambda \equiv c_0/f = \lambda$ (m) ≈300/f(MHz), substituting λ (m) by the frequency f/(299 792 458), the free-space propagation (dB) equations are:

$P_l\,(dB) = 20\ log \frac{4\pi d \times f}{299{,}792{,}458}$ =*20 log* 4π/299 792 458 + *20 log* d *(m)* + *20* log f (Hz)

$$P_l\left(\text{dB}\right) = -147.55 + 20\ log\ d\ \left(\text{m}\right) + 20\ log\ f\ \left(\text{Hz}\right) \tag{5.4.32}$$

Expressing d in km and f in MHz, (5.4.32) is the free-space equation at RR AP8-9 (the value 32.45) and Recommendation ITU-R P.525 (the value 32.4).

$$P_l\left(\text{dB}\right) = 32.45 + 20\ log\ d\ \left(\text{km}\right) + 20\ log\ f\ \left(\text{MHz}\right) \tag{5.4.33}$$

Next section details more formulas of free-space path loss; see equations(5.4.30) to (5.4.33) The free-space propagation loss; they specify that the attenuation depends **only** on the ratio d/λ. Firstly, the spreading out of the electromagnetic energy in free-space is determined by

the inverse square law, which is not a frequency-dependent effect. The wavelength (and frequency) dependence is derived from the receiving antenna's aperture, which describes how well an antenna can pick up power from the incoming electromagnetic wave. The received free-space field-strength is independent of frequency or wavelength, as the equation reflects only the spreading of wave. The received power at the antenna input is higher as the wavelength[23] is longer: for an isotropic antenna (Gain=0 dBi) the effective area equals $\lambda^2/4\pi$; see equation (5.4.13).
So, lower frequencies benefit from reduced propagation loss. It explains why the broadcasting medium waves (modulated AM) are received for longer distance relative to the broadcasting VHF (modulated FM), and why the VHF TV is received further than the UHF TV (for the same receiving antenna gains).

5.6.2.1 Friis equation to calculate radar equation in free-space

An interesting application of a free-space is the special case of Radar. The RF signal is subjected to a loss while propagating both from the transmitter to the target and back from the target to the receiver. The radar free-space basic transmission loss, p_l, in the common case where the transmitter and the receiver are at the same location, can be derived similarly as the *Friis transmission equation*; see equation (5.4.29).
Using standard international units, for:

p_t	=radar transmitter power	(W)
g_t	=radar transmitter antenna gain	(dimensionless)
d	=observation distance from the radar to the target	(m)
pd_{tg}	= incident power-density at the target	(W/m^2)
p_{tg}	= scattered power by the target	(W)
σ	= target RCS (Radar Cross Section)[24]	(m^2)
pd_r	=power-density at the radar receiver	(W/m^2)
p_r	=power received by the radar	(W)
A_e	= effective area of the antenna radar receiver	(m^2)
g_r	=radar receiver antenna gain	(dimensionless)

Using Friis equation, see (5.4.27): $pd_{tg}=\dfrac{p_t \times g_t}{4\pi d^2}$; $p_{tg}= pd_{tg} \times \sigma = \dfrac{p_t \times g_t \times \sigma}{4\pi d^2}$;

$$pd_r = \frac{p_{tg}}{4\pi d^2} = \frac{p_t \times g_t \times \sigma}{4\pi d^2 \times 4\pi d^2} = \frac{p_t \times g_t \times \sigma}{(4\pi d^2)^2}$$

$p_r=$ $pd_r \mathrm{x} A_e = \dfrac{p_t \times g_t \times \sigma}{(4\pi d^2)^2} \times A_e$; as $A_e = \dfrac{g_r \lambda^2}{4\pi}$, see equation (5.4.11);

$$p_r = \frac{p_t \times g_t \times \sigma}{(4\pi d^2)^2} \times \frac{g_r \lambda^2}{4\pi} .$$

Given that the radar (not the case of bi-static radars) uses a common antenna (mono-static) to transmit and to receive,

[23] And the frequency which equals about 300 MHz/wavelength (metre) is smaller.
[24] RCS is the ratio of the total isotropically equivalent scattered power to the incident power-density; ratio of power scattered by the target to the power-density incident on the target: $\dfrac{p_{tg}}{pd_{tg}}$.

$g_t = g_r = g$ and $p_r = \dfrac{p_t \times g \times \sigma}{(4\pi d^2)^2} \times \dfrac{g\lambda^2}{4\pi} = \dfrac{p_t \times g^2 \times \sigma}{(4\pi d^2)^2} \times \dfrac{\lambda^2}{4\pi} = p_t \times g^2 \times \sigma \dfrac{\lambda^2}{(4\pi)^3 d^4}$

Again, similar to the analysis to derive equation///, the radar free-space path loss equals the ratio p_t to p_r:

$$p_l = \frac{p_t}{p_r} = \frac{p_t}{p_t \times g^2 \times \sigma \dfrac{\lambda^2}{(4\pi)^3 d^4}} = \frac{(4\pi)^3 d^4}{g^2 \times \sigma \times \lambda^2} \qquad \textbf{(5.4.34)}$$

Again, to calculate the free-space path loss we may refer to isotropically antenna $g_t = g_r = g = 1$; and after replacing λ by c_0/f, the radar free-space path loss looks[25]:

$$p_l = \frac{(4\pi)^3 d^4}{\sigma \times \lambda^2} = \frac{(4\pi)^3 d^4 c_0^2}{\sigma \times f^2} \qquad (5.4.35)$$

The radar free-space path loss expressed logarithmically:

$$P_l \text{ (dB)} = 10 \log p_l = 10 \log \frac{(4\pi)^3 d^4}{\sigma \times \lambda^2} = 32.98 + 40 \log d - 10 \log \sigma - 20 \log \lambda \qquad (5.4.36)$$

After replacing λ by c_0/f

$$P_l \text{ (dB)} = 10 \log \frac{(4\pi)^3 d^4 \times f^2}{\sigma \times c_0^2} = 40 \log d + 20 \log f - 10 \log \sigma - 136.56 \qquad (5.4.37)$$

Changing the standard units to f (MHz) and d (km):

P_l(dB)=40log$d$+120+20log f+120–10logσ–136.56 = 103.44 +40log$d$ +20log f – 10log σ

The free-space assumption guides to the last formulas. If an attenuation of a (dB) is added to the free-space loss, a value of 2xa (dB) is added to the logarithmic propagation loss for radar.

If the target has a velocity component along the line-of-sight of the radar, the signal scattered from the target differs from the transmitter's frequency; the Doppler shift depends on the velocity, range rate $\dot{r}$, toward the transmitter; one way Doppler shift

$$\text{one way Doppler shift } f_D \approx -\frac{\dot{r}}{\lambda}, \text{ two way } \textit{Doppler shift } f_D \approx -2\frac{\dot{r}}{\lambda} \qquad (5.4.38)$$

The Doppler shift is positive when the source and the receiver are moving towards each other. Replacing λ by c_0/f, where c_0 is light velocity and f_t the transmitter frequency

$$\text{one way Doppler shift } f_D \approx -f_t \frac{\dot{r}}{c_0}, \text{ two way Doppler shift } f_D \approx -2 f_t \frac{\dot{r}}{c_0} \qquad (5.4.39)$$

[25] Losses due to antenna polarization, attenuations and atmospheric absorption are not included in this equation.

5.6.3 Maxwell's equations and received free-space field-strength from a far-field emission

5.6.3.1 Maxwell's equations

Four equations developed by Maxwell in 1863 present the classical electromagnetic theory at the macroscopic level. The equations relate time and space rates of change of various field quantities at a point in space. When working in phasor form, the notation of the sinusoidal time variation e^{jwt} is suppressed and jwt denotes its derivative. For the time-periodic case, when we are interested only how the wave propagates, and the wave is in a simple non-conducting media (a source free, linear, isotropic, homogeneous region), these are the four Maxwell's equations in the differential form[26]:

$$\nabla \times \vec{e} = -jw\mu\vec{h} \quad (5.4.40)$$

$$\nabla \times \vec{h} = jw\varepsilon\vec{e} \quad (5.4.41)$$

$$\nabla \cdot \vec{e} = 0 \quad (5.4.42)$$

$$\nabla \cdot \vec{h} = 0 \quad (5.4.43)$$

The two curl Maxwell equations in phasor form equations (5.4.37) and (5.4.38) constitute two formulas with two unknowns- the electric field vector $\vec{e}$ and magnetic field $\vec{h}$ (Pozar 2011 p.14-16). To solve $\vec{e}$, taking the curl (the mathematic function is ∇) of(5.4.37) and using (5.4.38) gives:

$$\nabla \times \nabla \times \vec{e} = -jw\mu\nabla \times \vec{h} = w^2\mu\varepsilon\vec{e} \quad (5.4.44)$$

$\nabla \times \nabla \times \vec{e} \equiv \nabla(\nabla \cdot \vec{e}) - \nabla^2\vec{e}$; $\nabla(\nabla \cdot \vec{e})$ =0, in a source free region;
therefore, the wave or homogeneous Helmholtz equation for $\vec{e}$ is:

$$\nabla^2\vec{e} + w^2\mu\varepsilon\vec{e} = 0 \quad (5.4.45)$$

The electric and magnetic fields have much similarity in Maxwell equations; an identical Helmholtz equation for $\vec{h}$ can be derived in the same manner:

$$\nabla^2\vec{h} + w^2\mu\varepsilon\vec{h} = 0 \quad (5.4.46)$$

The constant $k \equiv w\sqrt{\mu\varepsilon}$ is called the wavenumber, or propagation constant of the medium; its units are 1/m; its magnitude = 2π/λ where λ is the wavelength in m; see (5.4.48). The permeability μ and permittivity ε are the product of the free-space permeability μ_0 and relative permeability μ_r, and perfect permittivity ε_0 and relative permittivity ε_r respectively; therefore, $\mu = \mu_r\,\mu_0$ and $\varepsilon = \varepsilon_r\,\varepsilon_0$.

5.6.3.2 Plane waves in a lossless medium

In a lossless medium (free-space) μ, ε and k are real numbers. For a uniform plane wave solution, consider that $\vec{e}$ propagates in the z direction with no variations in the x and y directions: $\partial/\partial x = \partial/\partial y = 0$. Helmholtz equation (5.4.42) for $\vec{e}$ reduces to:

$$\frac{\partial^2 \overrightarrow{e_x}}{\partial z^2} + k^2\overrightarrow{e_x} = 0 \quad (5.4.47)$$

For two arbitrary real amplitudes e^+ and e^-, the solutions to (5.4.47), the time harmonic case at frequency w is:

$$e_x(z) = e^+(e^{-jkz}) + e^-(e^{-jkz}) \quad (5.4.48)$$

[26] The vector operator ∇ (pronounced del) denotes the partial derivative.

The time domain solution of equation (5.4.48) is:

$$e_x(z,t) = e^+(\cos wt - kz) + e^-(\cos wt + kz) \quad (5.4.49)$$

$e^+(\cos wt - kz)$ denotes a wave travelling in the +z direction, and $+e^-(\cos wt + kz)$ denotes a wave travelling in the -*z* direction. For a particular point (particular phase) on the positively travelling component, we set *cos (wt- kz)* constant, or *(wt- kz*) constant phase (Cheng p.308). The phase velocity of the wave is the derivative of the distance (not the phase derivative) in the *z* axis:

$$v_p = \frac{dz}{dt} = \frac{w}{k} = \frac{1}{\sqrt{\mu\varepsilon}} \text{ ; in free-space } v_p = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \equiv c_0 \quad (5.4.50)$$

c_0 ,the speed of light, also equals $\frac{w}{k}$. Equation (5.4.50) is an interesting formula; it relates the vacuum electric constant ε_0, vacuum magnetic constant μ_0 and the velocity of light c_0. The wavelength λ can be defined as the distance between two successive maxima at a fixed instance of time. The difference [*wt*-*kz*] – [*wt*-*k*(*z*+ λ)] equals *k* λ; the phase difference is one cycle: 2π radians in one period; as 2πf =w:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c_o}{w} = \frac{c_o}{f} \quad (5.4.51)$$

By symmetry to equation (5.4.48), for z_0 the wave impedance of the plane wave, the constant of the medium, we get in free-space:

$$h_y = \frac{1}{z_0}\left[e^+(e^{-jkz}) + e^-(e^{-jkz})\right] \quad (5.4.52)$$

The electric field and magnetic field in a lossless medium are in-phase.

$$z_o \equiv \frac{|\vec{e}|}{|\vec{h}|} \equiv \frac{w\mu_0}{k} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \mathbf{(5.4.53)}$$

5.6.3.3. Calculating free-space intrinsic impedance

z_0 is the ratio of $\vec{e}$ electric field and $\vec{h}$ magnetic field in a single travelling wave. $\vec{e}$ and $\vec{h}$ are orthogonal to the direction of propagation +- z ; it is a transverse electromagnetic (TEM) wave. z_0 depends on the free-space permeability μ_0 and free-space permittivity ε_0. In a lossless medium μ_0 and ε_0 are real numbers. The permeability of free-space μ_0, the magnetic constant, commonly called the vacuum permeability, equals by definition 4 π x 10^{-7} (henry/metre); the permittivity of free-space ε_0 (farad/metre), electric constant, commonly called the vacuum permittivity, is then derived from μ_0 and the velocity of light c_0 (m/s). Given c_o , velocity of light 299 792 458 m/s ≈ 300 10^8 m/s, the following equations calculate ε_0 and z_0. Based on equation (5.4.50):

$$\varepsilon_0 \equiv \frac{1}{c_0{}^2 \times \mu_0} \equiv \frac{1}{c_0{}^2 \times 4\pi 10^{-7}} \approx \frac{1}{(300\times10^8)^2 \times 4\pi10^{-7}} \approx \frac{10^{-9}}{36\pi} \quad (5.4.54)$$

$\varepsilon_0 \approx \frac{10^{-9}}{36\pi}$ (farad/metre); approximately 8.854 x 10^{-12} (farad/metre).

Given μ_0 equals 4 π x 10^{-7} (henry/metre) and c_o approximates 300 10^8 m/s:

$$z_0 \equiv \mu_0 \times c_0 \equiv 4\pi 10^{-7} \times c_0 \approx 4\pi 10^{-7} \times 3\times10^8 \approx 120\pi \quad \mathbf{(5.4.55)}$$

z_0≈120 π (ohms), approximately 377 (ohms). z_0 is needed to retrieve the electric and magnetic fields from the Poynting vector.

5.6.3.4 Poynting vector

The Poynting[27] vector $\vec{s}$ is the power-density (W/m^2) vector associated with the electromagnetic field (W/m^2); $\vec{s}$ is the vector (or cross) product of $\vec{e}$ (electric field) cross $\vec{h}$ (magnetic field). For the instantaneous vectors:

$$\vec{s} \equiv \vec{e} \times \vec{h} \qquad \textbf{(5.4.56)}$$

$\vec{s}$ is the complex power flow out of the closed area. The average power-density equals[28]:

$$s_{av} = \frac{1}{2}\mathrm{Re}(\vec{e} \times \vec{h}^{*}) \qquad (5.4.57)$$

The ½ factor appears in (5.4.57), as $\vec{e}$ and $\vec{h}$ fields represent peak values, the average refers to root mean square (rms) values; see Balanis p.14. $d\vec{a}$ is a vector normal to the surface: the infinitesimal area of the closed surface; the scalar-product of $\vec{s}$ (W/m^2) and $\vec{a}$ (m^2) is $p = (\vec{s} \bullet \vec{a})$ (W): the scalar power. The average power radiated by the antenna P_{rad} (radiated power)[29],[30]:

$$p_{rad} = \frac{1}{2}\oiint_a \mathrm{Re}(\vec{e} \times \vec{h}^{*}) \bullet da \qquad (5.4.58)$$

In free-space, the peak power-density s_{peak} in the main beam equals $\frac{e.i.r.p.}{4\pi d^2}$; see Friis equation (5.4.27). Moreover, as $\vec{e}$ and $\vec{h}$ are orthogonal, $|\vec{e} \times \vec{h}| = |\vec{e}| * |\vec{h}|$, the peak absolute value $|s_{peak}|$ is the product of the absolute values of $\vec{e}$ and $\vec{h}$. Using(5.4.27), the power-density equals:

$$|\vec{s}| = |\vec{e}| * |\vec{h}| = \frac{eirp}{4\pi d^2} \qquad (5.4.59)$$

For z_0 the intrinsic impedance of free-space (Ω, ohms), the time average of the instantaneous power flow per unit area, the Poynting vector (and power-density) is calculated after expressing the magnetic field-strength, see equation (5.4.53):

$$\vec{h} = \frac{\vec{e}}{z_o} \qquad (5.4.60)$$

$$|\vec{s}| = \frac{e^2}{z_o} = z_o h^2 \qquad (5.4.61)$$

Inserting $z_0 \approx 120\,\pi$ (ohms) from equation (5.4.55), the Poynting vector peak power-density approximates:

$$|\vec{s}| = \frac{e^2}{120\pi} = 120\pi h^2 = \frac{e.i.r.p.}{4\pi d^2} \qquad (5.4.62)$$

---

[27] See the definition of Poynting vector (and $\vec{S}$) at Balanis 2008 p.13 equation 1.3 and Cheng p. 327 equation 8-60.

[28] $\vec{h}^{*}$ is the complex conjugate of $\vec{h}$

[29] The vector operation $\oiint_a$ denotes an integral over an area.

[30] In free-space, at the receiver, $\vec{s}$ and $\vec{a}$ are parallel; thus the scalar-product is the multiplication of the absolute values of Poynting vector and area: $|\vec{s}| * |\vec{a}|$

The peak field-strength ***e*** is related to the square root of equivalent isotropically radiated power (*e.i.r.p.*) at far-field *free-space*:

$$e = \frac{\sqrt{30 \times e.i.r.p.}}{d} \quad \textbf{(5.4.63)}$$

where numeric-units (non-logarithmic) are used:

e = electric field-strength (V/m)

e.i.r.p. = product of the power and the antenna gain relative to an isotropic antenna (Watts)

d = distance from emitter to receiver (m)

Equation (5.4.63) can be replaced by equation which uses practical units:

$$e(\text{mV/m}) = 173\frac{\sqrt{e.i.r.p.(\text{kW})}}{d(\text{km})} \quad (5.4.64)$$

Using E.I.R.P. as the 10 log (e.i.r.p.), the field-strength equation expressed logarithmically in **standard units** looks:

$$E = 20\log e = 10\log 30 + E.I.R.P. - 20\ \log d = 14.77 + E.I.R.P. - 20\ log\ d \quad \textbf{(5.4.65)}$$

For other units, the field-strength equation expressed logarithmically looks[31]:

$$E(\text{dB}\ \mu\text{V/m}) = 14.77 + 120 + E.I.R.P. - 20\ log d = 134.77 + E.I.R.P.(\text{dBW}) - 20\ log d(\text{m}) \quad \textbf{(5.4.66)}$$

$$E\left(\text{dB}\ \mu\text{V/m}\right) = 134.77 + E.I.R.P. - 20 log d(\text{km}) - 60 = 74.77 + E.I.R.P.\left(\text{dBW}\right) - 20 log d\left(\text{km}\right)$$

Using the effective radiated power (E.R.P.)[32] instead of E.I.R.P., the field-strength link looks:

$$E\left(\text{dB}\ \mu\text{V/m}\right) = 74.77 + E.R.P. + 10 log 1.64 - 20 log\ d = 76.91 + E.R.P.\left(\text{dBW}\right) - 20 log\ d(\text{km}) \quad (5.4.67)$$

Using E.R.P. dB kW (=E.R.P. dB W+30), thus 106.91 replaces 76.91 to get[33]:

$$E\left(\text{dB}\ \mu\text{V/m}\right) = 106.91 + E.R.P.\left(\text{dBkW}\right) - 20 log d(\text{km}) \quad (5.4.68)$$

5.6.3.5 Clarifying example- calculating electric field-strength

For video TV operating at 514 MHz, antenna gain 50 (17 dBi), power 1,500 Watts, the e.i.r.p. is 1,500 x 50 Watts; using equation (5.4.60) to get the free-space field-strength at 75 metre in the main beam:

$$e = \frac{\sqrt{30 \times e.i.r.p.}}{d} = \frac{\sqrt{30 \times 1{,}500 \times 50}}{75} = 20\ \text{V/m}.$$

Equation (5.4.63) can be used to calculate the E.I.R.P. based on free-space conditions, measuring E= 96 dB(μV/m), at 1 km:

96 dB(μV/m) = 74.77+ *E.I.R.P.*(dBW) –20 log d(km);

96 dB(μV/m) = 74.77+ *E.I.R.P.*(dBW)–0; *E.I.R.P.* (dBW)=21.23 dBW, equivalent to 136 Watts.

[31] Compare to ECC Recommendation (12)03 where $E(dB\frac{\mu V}{m}) = 134.8 + E.I.R.P. - 20\ log\ d$

[32] "product of the power supplied to the antenna and its gain relative to a half-wave dipole in a given direction" RR 1: 1.162. The gain of a half wave dipole in free-space relative to an isotropic radiator is 1.64 (2.15dB).

[33] Figures and tables of ITU-R Rec. P.1546 refer to field-strength dB(μV/m) for 1 kW *e.r.p.*; see later.

5.6.4 ITU-R 1546- propagation curves 30 MHz to 3 000 MHz

Recommendation ITU-R P.1546 is most useful to predict point-to-area for terrestrial services in the frequency range 30 MHz to 3,000 MHz; among others, GE-2006 agreement, ECC Rec. (01)01 and ECC Rec. (11)04 use P.1546 to analyse sharing. P.1546 is based principally on statistical analyses of experimental data. It provides propagation curves representing field-strength values for 1 kW effective radiated power (e.r.p.) at nominal frequencies of 100, 600 and 2, 000 MHz, respectively, as a function of various parameters; some curves refer to land paths, others refer to sea paths. Interpolation or extrapolation of the values obtained for these nominal frequency values are used to obtain field-strength values for any required frequency, using a specified method given. Transmitting/base antenna heights used for field-strength versus distance curves are given for values of 10, 20, 37.5, 75, 150, 300, 600 and 1,200 m. The propagation curves represent the field-strength values exceeded for 50%, 10% and 1% of time; the representative height of ground cover is 10 m; see the following Figure. For interference analysis usually the 50% curves are used; in some sharing cases, the interest is in large signal levels (low attenuation) that occur for only 5% or 1% of the time or less. Telecommunications need signal available for more than 99% of time; this information is missing in P.1546.
Recommendation ITU-R P.2001 provides a general purpose wide-range model to predict terrestrial path loss, due to both signal enhancements and fading over effectively the whole range of time percentages, from 0% to 100% of an average year. The model is particularly suitable for studies in which it is desirable to use the same propagation model, with no discontinuities in its output, for signals which may be either wanted or potentially interfering. The model covers the frequency range from 30 MHz to 50 GHz, and distances from 3 km to at least 1,000 km. The following Figure, source: P.1546 Fig. 12, depicts land paths propagation 600 MHz, sea path, 50% time.

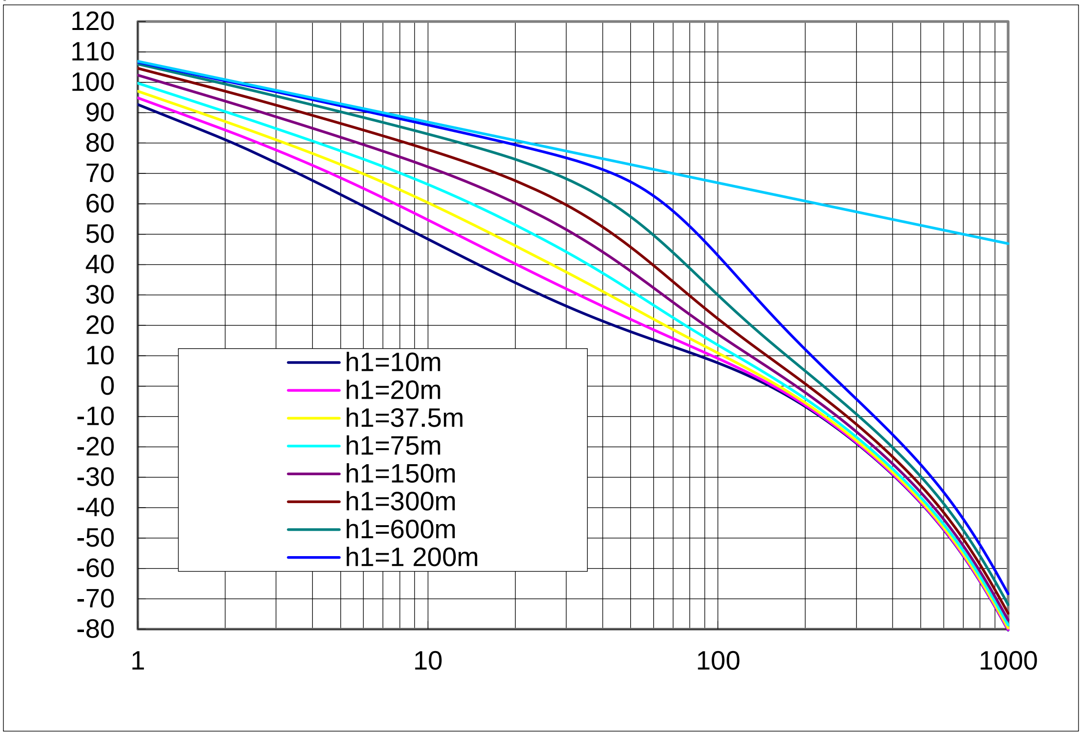

Figure 5.9 Field-strength (dB(µV/m)) versus distance (km) for 1 kW e.r.p. 600 MHz

5.6.5 Fresnel zones

Fresnel zones are important, as they classify which propagation model to use. The Fresnel zone is the ellipsoid that stretches between the transmitter and receiver antennas; A and B are the ellipsoid's focal points. The locus of points, such that the difference between the direct path $\overline{AB}$ and the indirect path $\overline{ACB}$ is half the wavelength λ (a phase change of π radians) of the transmitted signal; see Figure 5.10 Fresnel zone. F_n is the n[th] Fresnel zone radius; a difference of $n\frac{\lambda}{2}$, and phase change of $n\pi$ radians. When d is the distance between the two terminals; d_1 and d_2 are positive numbers such that $d_1 + d_2 = d$; see Figure 5.10 Fresnel zone . $d\ (\overline{AB}) = d_1\ (\overline{AP}\ ) + d_2\ (\overline{PB}\ )$. $\overline{PC}$ = Fresnel zone.

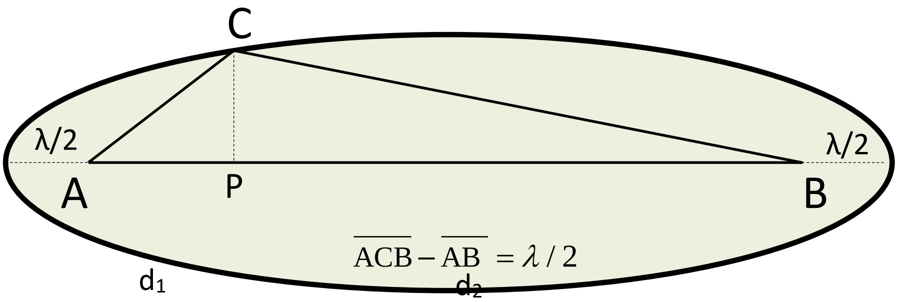

Figure 5.10 Fresnel zone

The radius of an ellipsoid, see the following Figure, at a point between the transmitter and the receiver F_n can be approximated in self-consistent units by:

$$F_1 = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}}\ ,\ F_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} \text{ and } F_n = \sqrt{n}\, F_1 \qquad (5.4.69)$$

Easy to show (when the derivative of $d(F_n)/d(d_1)=0$, that the maximum value of F_n appears at the middle of the path, where $d_1 = d_2$; $F_{n\_\max} = \sqrt{\frac{n\lambda d}{4}}$.

following equation (5.4.69), the third Fresnel zone $F_3 = \sqrt{\frac{3\lambda d_1 d_2}{d_1 + d_2}}$.

Using f frequency in GHz, d , d_1, d_2 path lengths in km, F_1 is the radius of the first Fresnel ellipsoid, in metres and F_3 is the radius of the third Fresnel ellipsoid, in metres.

$$F_1 = 17.3\sqrt{\frac{d_1 d_2}{f\, d}}\ ,\quad F_3 = 30\sqrt{\frac{d_1 d_2}{f\, d}} \qquad (5.4.70)$$

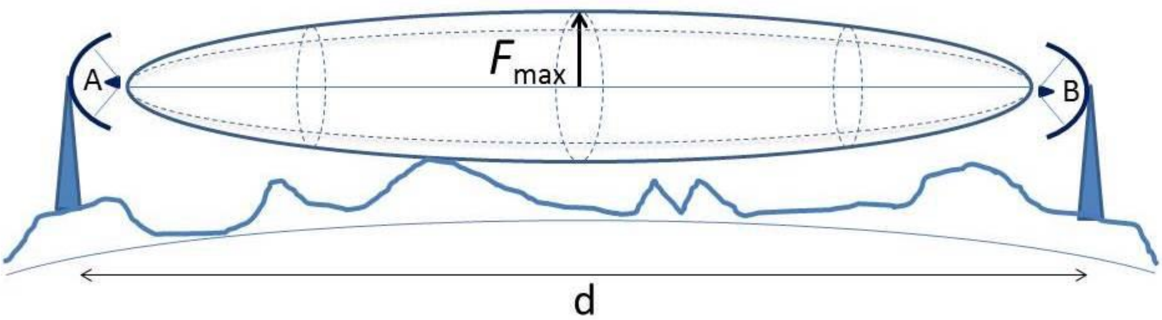

Figure 5.11 Fresnel ellipsoid: volume produced by rotating the ellipse around the ray AB

As a practical rule, propagation is assumed to occur in line-of-sight (LoS), i.e. with negligible diffraction phenomena, if there is no obstacle within the first Fresnel ellipsoid; see Rec. ITU-R P.526; Fresnel Zone is clear. If in the 1.5 to 3 Fresnel ellipsoids no obstructions exist, the free-space propagation model can be used.

5.6.5.1 Proof of Fresnel zones:

By definition, the difference of direct and reflected path equals n x (λ/2)

$$\sqrt{(d_1)^2+(F_n)^2}+\sqrt{(d_2)^2+(F_n)^2}-d=d_1\sqrt{1+\left(\frac{F_n}{d_1}\right)^2}+d_2\sqrt{1+\left(\frac{F_n}{d^2}\right)^2}-d=n\frac{\lambda}{2} \quad (5.4.71)$$

Using Taylor series expansion of a function about 0 (Maclaurin series), we get:

$$\sqrt{1+\left(\frac{F_n}{d_1}\right)^2}\approx 1+\frac{1}{2}\left(\frac{F_n}{d_1}\right)^2 \text{ and } \sqrt{1+\left(\frac{F_n}{d_2}\right)^2}\approx 1+\frac{1}{2}\left(\frac{F_n}{d_2}\right)^2 \text{ ; so}$$

$$d_1\sqrt{1+\left(\frac{F_n}{d_1}\right)^2}+d_2\sqrt{1+\left(\frac{F_n}{d_2}\right)^2}-d\approx d_1+\frac{1}{2}\frac{(F_n)^2}{d_1}+d_2+\frac{1}{2}\frac{(F_n)^2}{d_2}-d=\frac{1}{2}\frac{(F_n)^2}{d_1}+\frac{1}{2}\frac{(F_n)^2}{d_2}=n\frac{\lambda}{2}$$

(0.1)

$$\frac{(F_n)^2}{d_1}+\frac{(F_n)^2}{d_2}=n\lambda\text{ , }(F_n)^2=\frac{n\lambda d_1 d_2}{d_1+d_2}=\frac{n\lambda d_1 d_2}{d}\text{ and }F_n=\sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}\text{ ; }QED \quad (5.4.72)$$

5.6.6. Attenuation by atmospheric gases

The attenuation of atmospheric gases on terrestrial and slant paths can be computed by summation of individual absorption lines, that is valid for the frequency range 1-1,000 GHz, and estimating gaseous attenuation, that is applicable in the frequency range 1-350 GHz. Dry air attenuates the RF signal less than standard air. For earth-to-space applications, the atmospheric gases affect up to 30 km, and up to 100 km at the oxygen line-center frequencies. Recommendation ITU-R P.2001 calculates the sea-level specific attenuation for different types of radio path due to absorption by: oxygen, water vapour under non-rain conditions and water vapour under rain conditions. Figure 5.12 Specific attenuation in the range 50-70 GHz

, source: Recommendation ITU-R P.676 Fig. 2, depicts the specific attenuation in the range 50-70 GHz at the altitudes indicated. In higher altitudes the attenuation decreases, due to fewer gases.

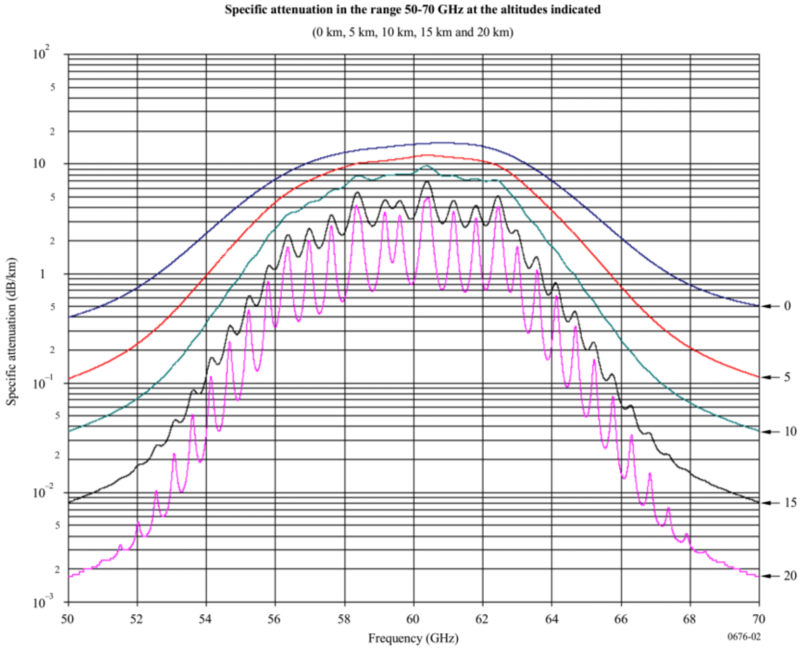

Figure 5.12 Specific attenuation in the range 50-70 GHz

For terrestrial applications the following Figure, source: P.676 Fig. 6, depicts the total, dry air and water-vapour zenith attenuation from sea level; pressure of 1,013 hPa (hecto Pascal, equal to 1 mbar), surface temperature 15°C and water- vapour density of 7.5 g/m^3. The attenuation, created by atmospheric gases (water vapour and oxygen) in those frequency ranges, may actually serve for secure (against listening) communications.

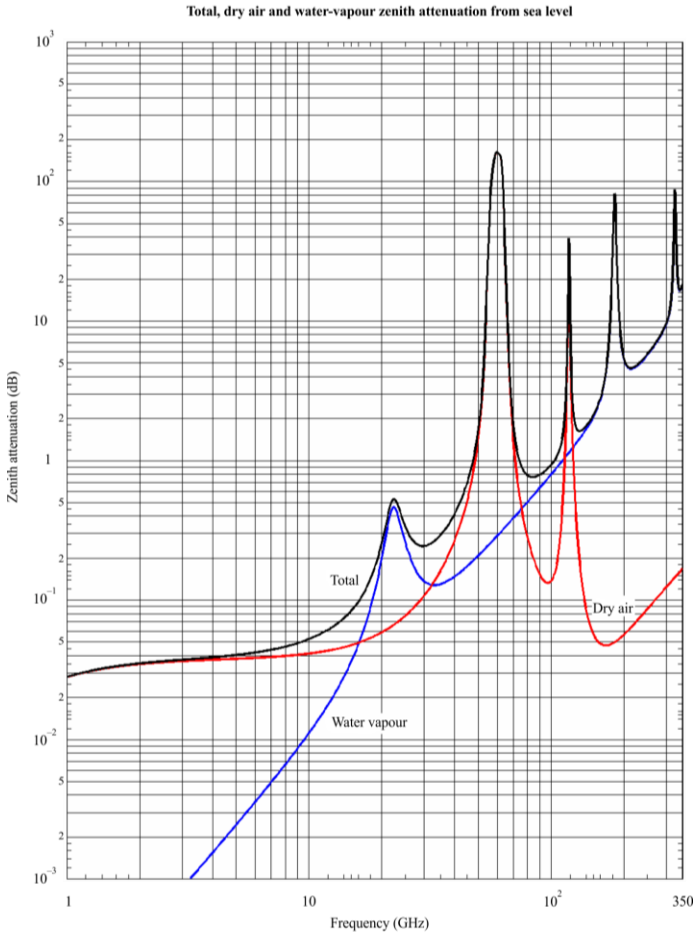

Figure 5.13 Specific attenuation in the range 50-70 GHz

5.6.7 Near-Field to Far-Field

Important to understand where the free-space model can be used; for example the difference in propagation between a handset cellular and the base station antenna; the first is near-field, and the second far-field. The space surrounding an antenna can be subdivided into three regions designating the field structure: reactive near-field, radiating near- field (Fresnel) and far- field (Fraunhofer). D the largest dimension of the radiator and λ the wavelength classify the regions. The far-field is 'that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna' (Balanis 2008 p.10). The Fraunhofer distance defines the far-field boundary of directive antennas; it equals $\mathbf{2\,D^2/\lambda}$, where the phase variation over the antenna aperture is π/8 radians (22.5^0). In the near-field, the electric and magnetic fields do not have a substantially plane-wave character. Reactive near-field is 'that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates' (Balanis 2008 p.10). A useful equation to calculate the reactive near-field of an antenna with maximum extension D is defined as maximum of λ, D and $\frac{D^2}{4\lambda}$. The radiating near-field (Fresnel) 'that region of the field of an antenna between the reactive near-field region and the far- field region wherein reactive fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna' (Balanis 2008 p.10). The boundary between the two near-field regions is $0.62\sqrt{\frac{D^3}{\lambda}}$. For small antennas, normally non-directive, the outer limit of near-field is between $\frac{\lambda}{2\pi}$ and 3λ.

5.6.8 Frequency dependency in penetrating walls and bypassing obstacles

Signals outside a building enter an enclosed building by penetration mostly through walls; wall penetration can also refer to the penetration through partitions inside buildings, see Recommendation ITU-R P.2040 Effects of building materials and structures on radiowave propagation above about 100 MHz. Higher frequencies are more sensitive to obstacles, land mobile stations (such as cellular) propagate better in lower frequencies; practically land mobile networks (excluding the fixed wireless between base stations) operate below 6 GHz[34]. This section tries to respond those significant questions: why the penetration in lower RF is better? Why the attenuation to in-door is higher when augmenting RF?

The ratio obstacle h versus wave length λ characterizes the three dominating phenomenon in radio waves propagation:

a) $h < \lambda$ ► diffraction ;

b) $h \sim \lambda$ ► scattering;

c) $h > \lambda$ ► reflection.

In lossless media (vacuum), $\mu_r = \varepsilon_r = 1$; $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ and $\sigma = 0$; $z_o \equiv \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi$ (Ω).

In a good conductor (metals, sea water…) the electric field decreases exponentially: the skin effect. Denoting the 'skin depth' as δ: the distance over which a plane wave is attenuated by a factor of e^{-1}; δ equals (see also Poazar p.18, equation 1.60):

$$\delta = \sqrt{\frac{2}{w\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu\sigma}} = \sqrt{\frac{\lambda}{\pi c_0 \mu\sigma}} \tag{5.4.73}$$

[34] See (the author's underline) the U.S. Presidential Memorandum 2013 '…to encompass more spectrum bands that may be candidates for shared access, specifically those in the range below 6 GHz….

For δ in metre, the attenuation rate in dB/m equals, see Recommendation ITU-R P.2040 equation 26:

$$Att_m = \frac{20 \log e}{\delta} = \frac{8.686}{\delta} \tag{5.4.74}$$

The signal attenuation in-door is related to building materials and RF. Given that c_o and μ are independent of frequency, and the value of σ is usually a strong function of frequency above 100 MHz, increasing with frequency: $\sigma = \alpha f^{\beta}$ (see P.2040 equations 28 and 87), the practical importance of the previous equation is that δ strongly increases with λ: lower frequencies (higher λ) benefit of better penetration and lower attenuation rate[35].

At higher frequencies, building entry losses, attenuation due to stone block wall and other obstacles increase; e.g., signals at RF lower than 6 GHz can penetrate through the metal doors of elevators. It explains why the broadcasting medium waves (modulated AM) are received in shelters and underground, contrary to VHF TV signals. The previous equation shows that even a thin layer of good conductor acts as an electromagnetic shield, only in higher frequencies; in lower RF the good metallic conductor does not prevent waves from radiating out of the shielded volume and do not prevent waves from penetrating into the shielded volume.

Lower RF profits better capability to by-pass obstacles. Given F_1, the radius of the first Fresnel ellipsoid as calculated, and h the height of the top of the obstacle above the straight line joining the two ends of the path[36] (h in the same units as F_1), Recommendation ITU-R P.530 specifies that the diffraction loss over average terrain can be approximated for losses greater than about 15 dB by the formula:

$$A_d = -20\, h / F_1 + 10 \quad \text{dB} \tag{5.4.75}$$

A_d increases with the RF as $F_1 = 17.3 \sqrt{\frac{d_1 d_2}{f\, d}}$ m. More generally, the attenuation due to an obstacle h is relative to the number of wavelengths $\frac{h}{\lambda}$; for the same obstacle h, the ratio increases with RF, as the obstacle comprises more wavelengths.

So, for penetrating walls, bypassing obstacles and practical propagation loss (versus distance-base station to portable), cellular systems utilize frequencies up to *circa* 6 GHz (centimetre waves). Present cellular carriers operate below 2,600 MHz; between 2,600 and 6 GHz they operate Wi-Fi, fixed and backhaul. 5G and 6G technologies are considering spectrum of up to 30 GHz for wireless communication, mainly for small cells.

[35] Polarization of the signal also influences the attenuation rate.

[36] If the height is below this line, h is negative; h is also the amount by which the radio path clears the earth's surface.

5.7 Link budget

5.7.1 Power equations

5.7.1.1 Link budget expressed numerically

Using the international system of units (SI), expressing p_r and p_t in the same units, the link budget equation calculates the received signal; expressed numerically it looks:

$$p_r = \frac{p_t \times g_t \times g_r}{p_l \times l_t \times l_r} \qquad \textbf{(5.4.76)}$$

where:

p_r = received power	(Watts)
p_t = transmitter output power	(Watts)
g_t = transmitter antenna gain	(dimensionless, with no units)
g_r = receiver antenna gain	(dimensionless)
l_t = transmitter losses (coax, connectors...)	(dimensionless)
p_l = propagation loss	(dimensionless)
l_r = receiver losses (coax, connectors...)	(dimensionless)

For the case of *free-space* transmission loss, where d (distance) and λ (wave length) are expressed in the same unit, the propagation loss p_l , see equation (5.4.30), and received signal p_r look:

$$p_l = \left(4\pi d / \lambda\right)^2 \qquad \textbf{(5.4.77)}$$

$$p_r = \frac{p_t \times g_t \times g_r}{\left(4\pi d / \lambda\right)^2 \times l_t \times l_r} \qquad (5.4.78)$$

5.7.1.2 Link budget expressed logarithmically

The link budget is mainly expressed logarithmically[37]; generally it looks:

$$P_r = P_t + G_t - L_t - P_l + G_r - L_r \qquad \textbf{(5.4.79)}$$

where:

P_r = received power		(dBW)
P_t = transmitter output power		(dBW)
G_t = transmitter antenna gain	(dBi)	
L_t = transmitter losses (coax, connectors...)		(dB)
P_l = Propagation loss		(dB)
G_r = receiver antenna gain	(dBi)	
L_r = receiver losses (coax, connectors...)		(dB)

For the case of **free-space** propagation, see equation// **Error! Reference source not found.**, using same units for d and λ (wave length) the transmission loss looks:

[37] Decibel (dB) indicates the ten times logarithm to base 10 of the ratio of two powers; e.g. the power p_1 referenced to p_0 can be set defined as 10 log10 (p_1/p_0). dBW is ten times logarithm to base 10 of the power referenced to one Watt; dBm is ten times logarithm to base 10 of the power referenced to one milliwatt (mW); p_1 (mW) is expressed in dBm as 10 $\log_{10}$ (p_1/1mW). dBi indicates gain of an antenna compared to the isotropic antenna: uniformly distributing power in all directions. Decibel (dB) also indicates the 20 times logarithm to base 10 of the ratio of two voltages, electric and magnetic fields; thus, as voltages, electric and magnetic fields derived from the square root of power. The rate in dB between two powers, relevant voltages or relevant field-strengths is equal; even that the voltage and field-strength rates are square root of relevant power rate.

$$P_l(\text{dB}) = 20\ log(4\pi d/\lambda) \quad \textbf{(5.4.80)}$$

Specifying the term 20 log 4π as 21.98, the free-space loss as function of *d/λ*:

$$P_l(\text{dB}) = 20\ log\ (4\pi d/\lambda) = 21.98 + 20\ log\ (d/\lambda) \approx 22 + 20\ log\ (d/\lambda) \quad \textbf{(5.4.81)}$$

$$P_r = P_t + G_t - L_t + G_r - L_r - 20\ log\ (4\pi d/\lambda) \quad \textbf{(5.4.82)}$$

Substituting λ by the f/c_0, (c_0 velocity of light in vacuum); the free-space propagation equations for f (Hz)=f (MHz) x10^{-6} are:

$$P_l(\text{dB}) = 20\ log \frac{4\pi d \times f(\text{MHz}) \times 10^{-6}}{c_0} = 20\log 4\pi/299792458 + 20\log d\ (m) + 20\log f\ (\text{MHz})$$

$$P_l(\text{dB}) = -27.55 + 20\ log\ d\ (\text{m}) + 20\ log\ f\ (\text{MHz}) \quad \textbf{(5.4.83)}$$

60 dB are added when using the unit (km) for distance:

$$P_l(\text{dB}) = 32.45 + 20\ log\ d\ (\text{km}) + 20\ log\ f\ (\text{MHz}) \quad \textbf{(5.4.84)}$$

Additional 60 dB are added when using the unit (GHz) for frequency:

$$P_l(\text{dB}) = 92.45 + 20\ log\ d\ (\text{km}) + 20\ log\ f\ (\text{GHz}) \quad \textbf{(5.4.85)}$$

Inserting the free-space propagation loss d (km) and f (MHz) to the received signal, we get the popular formula see equation:

$$P_r = P_t + G_t - L_t + G_r - L_r - 20\ log\ d\ (\text{km}) - 20\ log\ f\ (\text{MHz}) - 32.45 \quad \textbf{(5.4.86)}$$

5.7.1.3 Examples- link budget

For a **point-to-point** system, distanced d 40 km, operating at 7,500 MHz (λ wavelength 0.04 m), transmitting 2 Watts (33 dBm), 44.5 dBi (28,000) gains for transmitting and receiving antennas, 1 dB (1.25) cable losses at transmitter and receiver, assuming free-space loss $(4\pi d/\lambda)^2$, equivalent to 20 log (4πd/λ) (dB):

$$p_r = \frac{2 \times 28,000 \times 28,000}{(4 \times \pi \times 40,000 \div 0.04)^2 \times 1.25 \times 1.25} = 6.35\ 10^{-6}\ \text{Watts}$$

In logarithmic expressions, using the free-space loss 22 +20 log (d/λ), $P_l = 22 + 20\log \frac{40,000}{0.04} = 22 + 120 = 142$ dB and $P_r = 33 + 44.5 – 1 – 142 + 44.5 – 1 = –22$ dBm.

For **typical geostationary satellite downlink budget**, inserting the same values as in Table 5.1, d = 35,786 km, Ku Band link at f = 12.5 GHz, E.I.R.P. (Equivalent Isotropically Radiated Power) space to earth 52dBW equivalent to 82 dBm, earth station antenna (9 metre Diameter) gain 60 dBi. In logarithmic expressions, using the for free-space loss equation: P_l = 92.45 +20 log d (km) + 20 log f (GHz) = 92.45 + 91.07 + 21.94= 205.46 dB. Thus, the received logarithmic power P_r = 82– 205.46 + 60=-63.44 dBm; see ITU Handbook on Satellite Communications pp. 73-4.

5.7.2 Conversion formulae

5.7.2.1 Received power, power-density and field-strength

Equation (5.4.56), stating the Poynting vector $\vec{s}$, can be used to calculate the power-density, $\vec{s} = \frac{e^2}{120\pi}$; the relation power-density and field-strength is independent of RF. The following equations are adequate at far-field free-space; however, these equations approximate other conditions. The equations are useful, also in the near-field; the usual case for RF, lower than 30 MHz.

The power-density expressed logarithmically looks:

$$S\ \left(\text{dB W/m}^2\right)\ =\ E\ \left(\text{dB V/m}\right) - \ 25.76 \tag{5.4.87}$$

A useful equation to calculate the broadcasting minimum median field-strength level expressed in dBμV/m:

$$S\ \left(\text{dB W/m}^2\right)\ =\ E\ \left(\text{dB μV/m}\right) - \ 145.76 \tag{5.4.88}$$

As the effective antenna aperture of the receiver A_e equals $\frac{g_r\lambda^2}{4\pi}$, see equation (5.4.11), the received signal equals power-density $\vec{s}$ multiplied by the effective antenna aperture A_e:

$$p_r = \frac{e^2}{120\pi}\times\frac{g_r\lambda^2}{4\pi} \tag{\textbf{5.4.89}}$$

As c_o (velocity of light) ≡ 299 792 458 m/s ≡λ x f; after expressing λ by f (MHz),

$$p_r(\text{W}) = \frac{e^2}{120\pi}\times\frac{(c_o)^2\times g_r}{4\pi f^2(\text{MHz})10^{-12}} = \frac{89{,}875.518\times e^2\times g_r}{480\pi^2 f^2(\text{MHz})} = \frac{18.97\times e^2(\text{V/m})\times g_r}{f^2(\text{MHz})} \tag{0.2}$$

p_r can be expressed **logarithmically** in dBW, in standard units, except f (MHz):

$$P_r(\text{dBW}) = 10\log 18.97 + 20\log e\ (\text{V/m}) + G_r(\text{dBi}) - 20\log f\,(\text{MHz})$$

$$P_r\left(\text{dBW}\right)\ =\ 12.78\ +\ E\ \left(\text{dB V/m}\right)\ +\ G_r\left(\text{dBi}\right)\ -\ 20\ log\ f\ \left(\text{MHz}\right) \qquad (5.4.90$$

Rearranging the above, the free-space field-strength is calculated from the received power with G_r gain as follows:

$$E\ \left(\text{dB V/m}\right) = \ 20\ log\ e\ \left(\text{V/m}\right)\ =\ P_r\left(\text{dBW}\right) - 12.78 - G_r\left(\text{dBi}\right)\ +\ 20\ log\ f\ \left(\text{MHz}\right) \tag{5.4.91}$$

If E(dB(μV/m)) and P_r (dBm): E(dB(μV/m))–120=P_r (dBm)–30–12.78–G_r(dBi)+20log f (MHz)

$$E\ \left(\text{dB μV/m}\right) = \ 77.22 + \ P_r\left(\text{dBm}\right) - G_r\left(\text{dBi}\right)\ +\ 20\ log\ f\ \left(\text{MHz}\right) \tag{\textbf{5.4.92}}$$

The conversion of received power and magnetic field is typical to Short Range Devices (SRDs) operating below 30 MHz.

5.7.2.2 Received power, voltage and antenna factor

Power is related to voltage, according to the basic Ohm's law:

$$p_r = \frac{v^2}{r} \tag{5.4.93}$$

The logarithmic expression of Ohm's law for received power in a 50[38] Ω resistance load:

[38] Impedance of coaxial cables is usually either 50 ohms or 75 ohms: 50 Ω is primarily used for data signals in

$$P_r\left(\text{dBW}\right) = 20\ log\ v\ \left(\text{Volt}\right)\ -10\ log\ r\ \left(\text{Ohm}\right) = V\ -10\ log\ 50 \approx V - 17 \quad (5.4.94)$$

For power in dBm, V (dB μV) in 50 Ω resistance load, P_r (dBm) –30 ≈ V (dB μV) –120–17

$$P_r\left(\text{dBm}\ \right)\ \approx\ V\ \left(\text{dB}\,\mu\text{V}\right)\ -107 \qquad \textbf{(5.4.95)}$$

If the received power is calculated in a 50 Ω resistance load, the ratio e/v is derived from equation (5.4.89):

$$p_r = \frac{v^2}{50} = \frac{e^2}{120\pi} \times A_e = \frac{e^2}{120\pi} \times \frac{g_r\lambda^2}{4\pi} \qquad (5.4.96)$$

The antenna factor is the ratio of the incident electric field-strength to the voltage. Denoting the coefficient $\frac{e}{v}$ as the antenna factor a_f, for a free-space intrinsic impedance $z_0 \approx 120\ \pi$, see equation (5.4.53) and a load of 50 Ω resistance :

$$a_f = \frac{e}{v} = \sqrt{\frac{480\pi^2}{50 g_r \lambda^2}} = \frac{9.73}{\lambda\sqrt{g_r}}(\text{m}^{-1}) \qquad (5.4.97)$$

A_f is the antenna factor expressed **logarithmically**:

A_f =20 log ($\frac{e}{v}$) =E dB (V/m)–V dB (V) =E dB (μV/m)–V dB (μV)= 20 log 9.73–20 log λ–G_r

$$A_f\,(\text{dBm}^{-1}) = E(\text{dBV/m}) - V(\text{dBV}) = E(dB\mu\text{V/m}) - V(\text{dB}\mu\text{V}) = 19.76 - 20 log\lambda - G_r$$

(0.3)

As the effective antenna aperture from equation **Error! Reference source not found.** A_e = $\frac{g_r\lambda^2}{4\pi}$, $g_r\lambda^2$ equals $4\pi A_e$; inserting $4\pi A_e$ instead of $g_r\lambda^2 \cancel{A_e}$ at equation **Error! Reference source not found.**, we get:

$$a_f = \frac{e}{v} = \sqrt{\frac{480\pi^2}{50 g_r \lambda^2}} = \sqrt{\frac{480\pi^2}{50 \times 4\pi \times A_e}} = \sqrt{\frac{2.4 \times \pi}{A_e}} = \frac{2.75}{\sqrt{A_e}}(\text{m}^{-1}) \qquad (5.4.98)$$

and expressed in logarithm

$$A_f\ dB\ \left(\text{m}^{-1}\right)\ =\ 20\ log\left(a_f\right) = 20\ log\sqrt{\frac{2.4 \times \pi}{A_e}} = 8.78\ -\ 10\ log\ \left(A_e\right) \qquad (5.4.99)$$

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a two-way communication system; 75 Ω for television and video signals.

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